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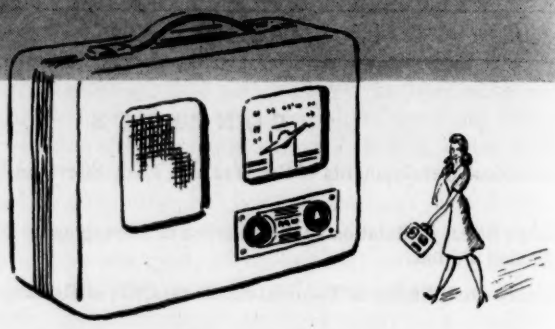

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Journal
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Vol. 240 AUGUST, 1945 No. 2

SOME TECHNICAL DEVELOPMENTS OF THE WAR AND THEIR EFFECTS IN PEACE.*

BY

RALPH E. FLANDERS, Sc.D., D.E.,

President (on leave) Jones & Lamson Machine Company, Springfield, Vermont; President, Federal Reserve Bank of Boston.

It is with some hesitation that one approaches a subject like this at a time like this. We have finished one grim and murderous war and are in the midst of another equally fierce and deadly. Why should we have any thoughts which are not directed toward bringing to a conclusion the business we have in hand?

The answer is that we have a responsibility for the post-war world. This responsibility cannot be shirked, nor can it be met by hasty improvisations at the war's end. We are continuing our miracles of production and transportation, with heavy expenditures of physical and nervous energy. At its highest pitch this is but a fraction of the burden laid on our sons overseas and they face the added perils of disease, wounds and death. It is because we owe them an unpayable debt that it is right for us to give thought to the post-war world.

In carrying out that commission the United Nations at San Francisco are endeavoring to prepare the blue-prints of a political world organization which shall help us to avoid such tragic conflicts, as this and the previous generation have endured. Here tonight we are a part of the more humble, but yet important task of considering the materia possibilities of the post-war world.

We have performed miracles of production, transportation, engineering and science. Our purpose requires that we make a cursory examination of these miracles.

* Presented under the Charles Day Lecture Foundation of The Franklin Institute of Pennsylvania, given in the Hall of The Franklin Institute, at the Stated Meeting held May 23, 1945.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

Our miracles of production have been astonishing. Aside from the skill and bravery of our armed forces, and the grim endurance of our English and Russian allies, these efforts of ours have been the decisive element in winning the European war, and will be likewise effective in bringing to a close our Asiatic operations. We have astonished not merely the world, but ourselves, by our output of ships and planes.

Some economists and statisticians have observed this phenomenon, examined some of the detailed methods by which it was accomplished, and have deduced therefrom that we have discovered new production secrets and will come out of this war with a substantially raised level of output per man-hour.

There have been innumerable new wrinkles devised in the manufacture and assembly of the materials of war, but that goes on continuously in peace time production. Has the process been accelerated? My own observation, for what it is worth, indicates that the improvement is essentially one in *war* production, with perhaps no more than the usual annual accretion applicable to the arts of peace.

Of the improvement which arrests the attention and makes the headlines, the major part is concerned with products quite new to the contractors and their employees. They have to learn to make them on a quantity basis, in the same way they would undertake any new production job. This development period is speeded up, it is true, by patriotism, limitless orders and the provision by the government of any needed working capital. In consequence the ten-thousandth airplane costs a small fraction of the tenth. There is nothing novel in this as an over-all process, though there may be in some details. It is simply a new example of that genius for production which has made America industrially great.

Another similar case is that of the product formerly required in small quantities which under war conditions meets an unmeasured demand. Ships are a good example. We had a number of experienced shipbuilders who built ships to order singly, or in lots of two or three, by conventional methods. Then came the Liberties and the Victories, and our flair for mass production put them almost on an assembly line basis. That demand will die with the war, leaving little more of salvage, perhaps, than the valuable experience with welded hulls.

Possibly the greatest advantage in production has been in the acceleration and larger use of practices already introduced. A good example is the expanded use of carbides as cutting tools in the place of high speed steel. The carbides themselves have been improved. Machine tools are as yet only beginning to be adapted to make full use of the new possibilities, which require speeds and power intake far beyond the standards of conventional practice. The thoroughgoing redesign of machine tools which followed and was made necessary by Taylor's invention of high speed steel must be duplicated for the

carbides. Very possibly the war has hastened or will hasten the process. But taking it all in all the chances for an increase of production as an after effect of the war seem rather thin. Many industrialists are in fact disturbed by the fear that man-hour production will actually be lower instead of higher. The managers of few automobile plants, for instance, hope for the same labor efficiency in resuming manufacture as was current before the shift to war production.

This possibility, and the certainty of higher wage rates, will force a new development of automatic labor-saving machinery in all the remaining operations to which it has not yet been fully applied. Only by this means can automobiles and other durable consumer goods be kept within such price limits that those who make them can afford to buy them—not to mention the rest of us. A feverish acceleration of cost reducing equipment is definitely in prospect.

In one field of production, definite advances have been made under the spur of war-time developments. The chemical industry has benefited from the immense demands made upon it. Continuous processes have replaced "batch" operation in oil refining, an improvement made possible by the development of continuously recoverable liquid catalysts. The application of automatic temperature, analysis, and other controls has been particularly effective in the manufacture of synthetic rubber. By enlarging operations and by applying automatic controls, early war sales prices of 18 cents for butadiene and 31 cents for styrene were reduced to a range between 6.6 cents per pound and 8.8 cents per pound in the one case and between 4.95 cents and 5.75 cents in the other. This is considered to be by no means the end of a cost reduction which should bring artificial rubber down to a point where it competes effectively in price with the imported natural article.

This situation raises serious questions of a long-range nature. Should we shift to the synthetic product, we will destroy the major part of the market of the tropical rubber-producing countries and of the natives who have become dependent on it for their livelihood. On the other hand, we will be using up our oil resources, of which the limit must sometime be reached in spite of the fact that at any time for some years past there has not been in sight more than fifteen years' reserves. Sometime, those reserves will be drawn on faster than new supplies are discovered. It is, of course, possible that improved technology will permit us to synthesize rubber from coal somewhere nearly as satisfactorily as we can do it from petroleum, in which case our 300 to 1,000 years' reserves of coal will not be so seriously threatened as are our oil reserves.

We must keep our synthetic rubber plants in working condition, but so far as concerns their being a main dependence for our source of raw materials for automobile tires, the decision must be made on high political grounds and on long-range policies of protection of

natural resources rather than on the immediate technical and commercial problems involved.

An important addition to our list of fuels is the new "triptane," whose octane number is so high that it is hardly worthwhile to calculate. It can be used in comparatively small quantities to be mixed with other lower grade fuels to bring their octane count up to 100 or to whatever other reasonable level may be desired. This has real importance in connection with the automotive fuels for peace-time.

Let us pursue further this subject of the miracles of engineering of which our production of synthetic rubber is but one.

Popular attention has been focused very largely on what may be called the "gadgets" of war-time engineering development. As an example of this direction, we find an article by Waldemar Kaempffert in this past week's Sunday Times which reviews technical progress made during the war which should help bring a high standard of living. The examples are drawn from 1,400 items listed in a 400-page report entitled "War-Time Technological Developments" and presented to Congress by the Subcommittee on War Mobilization, of which Senator Kilgore is Chairman. The list includes some things which are routine rather than new, such, for instance, as "a machine tool designed to drill thirty holes in an airplane engine crank case in 25 minutes." Every production man will recognize this multiple-spindle drill as an old friend.

More worthy candidates for attention are "self-cleaning nozzles" for certain spraying operations, an electronic device for detecting pinholes in metal strips and marking defective sections for rejection, a mobile oil refinery for purifying crank case oil, methods of reclaiming valuable substances from the waste liquors of paper mills, rivets of plastic, etc.

One of the items is prefabricated houses with packaged kitchens, bathrooms, and heating units, and durable plastic or plywood walls to be delivered and "welded" on the site by techniques now used in shipbuilding. Great progress was made on similar housing designs during the depression but the best information I was able to get as to what prevented the full development of the idea was that it had not been possible to clear the project with union labor in the building trades.

Most of the glimpses of the post-war world similarly concern themselves with gadgets. We have, for instance, the promise of telephones which will record a message when you are out and deliver it for you when you return. Electronic and electrical eye devices of various sorts are promised us. Television and deep freeze are due to a wide expansion. It is probable that more flat irons will be streamlined and that more of our steam locomotives will wear tin skirts. Many of these things are important; few of them have great significance other than as they are a

part of a continuous development, desirable as a whole, which has been arrested by the war rather than stimulated by it.

The war has made some rather important changes in the future of textiles. It is, for instance, quite possible that silk will become an undesired rarity, though women will determine this matter, rather than we men. All we have done has been to provide substitutes, like nylon, more and more nearly approaching the desirable characteristics of the natural silk. We do have in prospect a non-shrinking wool developed to meet Army needs. We have a new plastic water-proofing which may be applied to any textile, permits it to be washed or cleaned, and does not become porous with age. The English have produced, for use by desert armies, a new type of underwear which, in warm weather, is porous and opens up to permit the escape of perspiration. When cold night covers the desert, these pores close up and the underwear is tight and warm. While these improvements are comparatively small ones, yet they will add to the comfort and usefulness of the clothes we wear every day.

Great advances have been made in metallurgy. We are able to build new qualities out of old elements. We are able to use metals for alloying purposes which are in good supply to take the place of those which are in short supply and expensive. We can, in fact, duplicate the desired physical characteristics of one type of steel in other types in which the alloying elements are quite dissimilar.

This was of great importance early in the war when our resources of certain important alloying minerals were cut off. Our new inventions were born of dire necessity but will remain useful in peace-time. Among the steels the most important advances, perhaps, are those which have resulted in heat-resisting, corrosion-resisting, and erosion-resisting alloys for airplane engines, and particularly for gas turbines. But of this, more later.

Airplane design has, of course, been tremendously advanced by the war. We fly faster and higher and carry greater loads at less fuel cost by far than we were able to do in the early 'forties. As a consequence, in the post-war world the plane will compete successfully with the train for most of the long-distance traffic. So far as ships are concerned, London and Paris will be as near to us in time as Chicago and St. Louis are by train now, and we will eventually, though not immediately, adjust our habits of thought and of action to this new propinquity.

The same revolution, though on a larger scale, will obtain in the speed of our connections with the Far East. We can only dimly discern the commercial and political changes which this development will bring about.

So far as the transport of goods is concerned, there appears to be nothing in prospect to supplant land and water routes except for emergency and high-value shipments. In general, materials will still

continue to travel over the highways, the rails, and the surface of the sea.

The most radical developments in planes have, of course, been made in the last few months by the development of jet propulsion. This undertaking is only in its infancy. It has several important possibilities. While at present it is not economical in miles per gallon as compared with the older types of reciprocating internal combustion engines, there is good reason for believing that it will approach them in economy of fuel consumption as it is further developed. It has, at the moment, however, the great advantage that it does not require high octane fuel. Lower grade fuels such as kerosene, which cannot be used in an automobile, are perfectly satisfactory for jet propulsion. Improvements in the device and in the plane to which it is attached lead us to believe that we can no longer think of it as impossible to fly west, keeping pace with the sun as the earth rolls beneath us. We can conceivably breakfast in Philadelphia and reach San Francisco in time for breakfast on the same day. I am assured by those who should know that this possibility is not an idle dream.

The practicability of jet propulsion is dependent on many things, among them those developments in metallurgy of which we have just been talking. The efficiency of the prime mover depends, among other things, on a high compression of the charge of air and fuel which is to be ignited in the jet. This high pressure is obtained by a rotary compressor operated by a gas turbine in the jet exhaust. It is the development of metals which resist corrosion and erosion at high temperatures which made possible the turbine blades which work successfully in this apparatus.

Not only does jet propulsion drive a plane at higher speed and use less costly fuel, but it also is very much simpler and less expensive than an engine that is the conventional type of motor, whether radial or in-line. The improvement and successful application of this motor is primarily the improvement and successful application of a gas turbine. A successful prime mover of this sort has long been sought and through the medium of jet propulsion now appears to be on the way. Instead of burning oil under boilers and then running the steam through the turbine, it is quite probable that similar economies can be attained for power plants by burning the oil in the jet of a gas turbine directly, without having first to transfer the heat of combustion to water.

Further developments which can be safely contemplated are the use of gas turbines for railroad locomotives. In this case, coal would be burned in a producer-gas generator located on the tender. The producer-gas would then be used as fuel in the gas turbine of the locomotive which, by electrical or other transmission, would conduct the power to the driving wheels.

Here again we have an important consideration in the field of the

conservation of natural resources. The present shift from coal-burning locomotives to oil-burning Diesels is one that has everything in its favor so far as concerns economy in operation and maintenance and smoothness of operation. The only thing against it is that it is assisting, though at present only in a small way, in drawing down oil reserves. If successfully replaced by the producer-gas turbine, this threat to our future will be removed.

From the researches of pure science have come such developments as radar, which competent engineering has developed into useful and marvelous devices. From the first installation of radar at Pearl Harbor, which gave a warning of the attack of the Japanese which was unheeded, to the time in the later months of the war when planes were shot from the sky and cruisers shot in the dark of night by ships of whose existence the enemy was unaware until they were hit—from the early installation to the present-day perfection are represented many years of normal development compressed by the insistent demands of the war into a few months.

Radar and related radio developments are going to be of great use in peace-time. The earth will be covered with navigation grids which will enable the captain of a ship or the pilot of a plane to fix his position with assurance at any time, night or day, and in any weather. Collisions, even in the densest fog, in the air or on the sea, are going to become practically non-existent and inexcusable when they do occur. The same is true of collisions with icebergs and ice flow. Likewise, it is going to be possible to take off and land in planes from a properly equipped air field even though the pilot is unable to see a hundred feet ahead of him.

Not the least of the war-born marvels are those which medical science has provided. The sulpha drugs have had great development and improvement. The most remarkable advance, however, has come in the application of penicillin and in its production on a mass basis. Recoveries from wounds are made as a matter of course where they would have been hopeless a few years ago. Germ infections are arrested and the patient returned to full health.

In what is at first sight a more lowly field, new insect repellents and insecticides have added enormously to the comfort of life in the tropics. The improvement, however, is more than in mere comfort. It means a definite diminution in infection from insect bites, and the easier maintenance of health in hot and humid countries.

The whole area of tropical diseases is coming under medical control. In this field, we have the possibility of major changes in human history far more significant than the galaxy of gadgets which popular journalism presents as the fruits of war technology. In a few generations, civilized man, having drawn all too heavily on his mineral resources of coal, petroleum, iron, and other metals, will be replacing them with the

reproducible and indefinitely productible products of the soil. The centers of production for the raw materials of the new technology will doubtless be in the tropics and the regions bordering the tropics. It is of fundamental importance to this probable future of human development that we master by our science the art of living comfortably and healthfully in these hot and humid countries. In long retrospect, this may turn out to be the greatest change brought about by our war technology.

We have been considering the *wonders* of science and engineering. We would be inexcusably blind did we not give some thought to the *horrors* of science and engineering. All of the developments we have been considering tonight are the hopefully useful by-products of an enormous undertaking on the part of civilization to commit suicide. The purpose has been destructive to the extreme limit of human power, even though from the edges and the by-products of this Satanic endeavor we have culled a few fruits of human usefulness.

Were the whole story publicly revealed, we would discover that the possibilities of destructiveness are far from having been exhausted. We would discover that new inventions of destruction are ready for use should the fortunes of war require us to use them. From this, we may be assured that if in the lapse of time a new conflict arises, it will be on such a scale of cruelty and destruction as will dwarf the horrors to which we have become all too well accustomed in this conflict.

It is the conviction of your speaker that war is the inescapable result of wrong-doing on a national and international scale. The only possibility of avoiding war lies in learning to live together in justice and Christian brotherhood. It is this necessity and this threat which gives such deep urgency to such endeavors as are being made in these days at the San Francisco Conference to bring the nations of the earth to see their common interest and their common peril and to act and organize in accordance therewith.

It is not a matter of getting men to be brothers. Human brotherhood is wrongly viewed if it is considered as an ideal attainable only in the far distant future, if ever, but which we must struggle to attain. Human brotherhood, on the contrary, is a terrible existing fact which we cannot escape. It is the fact of this brotherhood which, as the result of regional and international wrong-doing, throws a world into murderous war.

This is the great lesson to be learned from six years of horror, and unless we have learned it, all other human knowledge is vain.

THE CHARGE EFFECT IN RELATION TO THE KINETICS OF PHOTOGRAPHIC DEVELOPMENT. II. THE INDUCTION PERIOD.

BY

T. H. JAMES,

Communication No. 1032 from the Kodak Research Laboratories.

The characteristics of the development process during the early stages depend markedly upon the nature of the developing agent, as well as upon the composition of the developer solution and the nature of the photographic material employed. The phenomenon of the induction period plays an important rôle in producing this variation. The influence of the induction period may be shown in several ways, but it is especially apparent in the shape of the lower portion of the curve obtained by plotting the density against the time of development. Figure 1 illustrates typical curves ranging from one showing no induction period (0) to one showing a marked induction period (-3).

The shape of the curve representing the initial stage of development is largely determined by the charge of the active ion, other things being equal, rather than by the chemical class to which the developer belongs. When development of a simple silver bromide emulsion is carried out in solutions of simple composition, the curves representing the initial stage fall into distinct groups characterized by the charge of the active ion. (This is clearly shown [1] by the published data.) Figure 1 gives a comparison of the curves obtained, according to whether the active developing agent was uncharged or was in the form of singly (-1), doubly (-2), or triply (-3) charged ions. The comparisons were made at equal concentrations of developing agents, in solutions containing small additions of sodium sulfite to minimize the action of oxidation products, and at pH values in the range 8.3–8.6 (except for the triply charged sodium hydroquinone monosulfonate, where the pH was 9.5). The photographic material was a normal motion-picture positive film (Emulsion "C"). In the construction of the figure, the time scale for each curve was adjusted (by using a suitable multiplier) to give approximately equal slopes to the straight-line portions of all the curves.

Under the simplified [1] conditions of development, the relative extent of the concave toe portion of the density-time curve depends not only upon the charge of the developing agent but also upon the magnitude of the bromide barrier. Bromide ion in the developer solution increases the bromide barrier; phenosafranin depresses it.[2] The effects of these

[1] T. H. James, *J. Phys. Chem.*, **43**, 701 (1939).

[2] Cf. Part I of this series, J. FRANK. INST., **240**, 15 (1945).

substances on the shape of the curve representing development by a typical doubly charged agent are illustrated in Fig. 2. Once more, the time scale has been adjusted for each curve so as to give equal slopes to the straight-line portions. This procedure supplies a "correction" for the change of rate beyond the toe region, and permits an easier comparison of the relative extents of the toe region under the different conditions. The singly charged agents show much less relative sensitivity to bromide in the early stages of development than do the doubly and triply charged agents. Curves representing development by

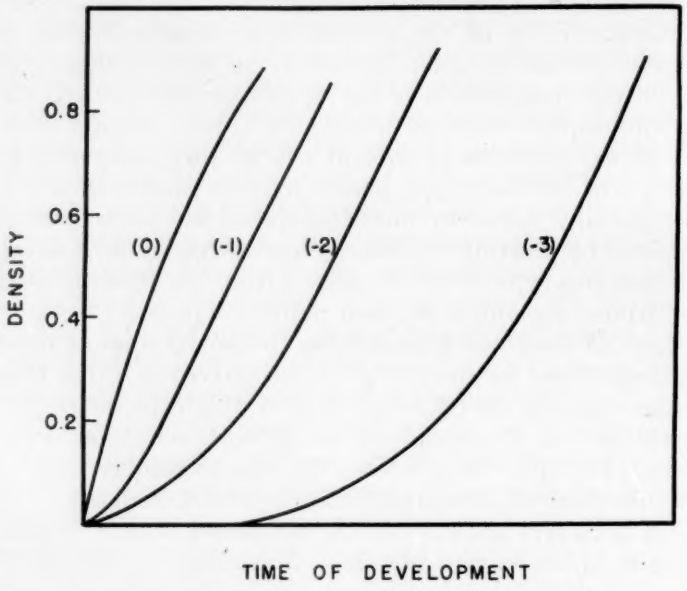

FIG. 1. Dependence of shape of development curve upon charge of developing agent. Normal motion-picture positive film "C" employed.

uncharged agents generally show no concave toe whatsoever, either in the presence or absence of bromide.

ORIGIN OF THE INDUCTION PERIOD.

Berg [3] has recently proposed an explanation of the induction period based upon the assumption that development takes place in two phases. The first phase, according to his view, is identical with that previously suggested by Gurney and Mott [4] as applicable to the entire development process. The latent-image silver speck is assumed to act as an electrode, and to receive electrons from the developing agent. Interstitial silver ions from the silver halide crystal are attracted to the silver

[3] W. F. Berg, *Trans. Faraday Soc.*, **39**, 115 (1943).
[4] R. W. Gurney and N. F. Mott, *Proc. Roy. Soc.*, **164A**, 151 (1938).

speck and there neutralized. According to Berg's view, any rapid development by this process quickly comes to an end as the supply of interstitial ions originally present in the crystal becomes exhausted, since new interstitial ions form at only a relatively low rate. If, however, the silver speck at that point is not large enough to initiate the second phase of development (which is assumed to proceed by a different mechanism), development continues slowly by the first mechanism, but only at the rate of formation of new interstitial silver

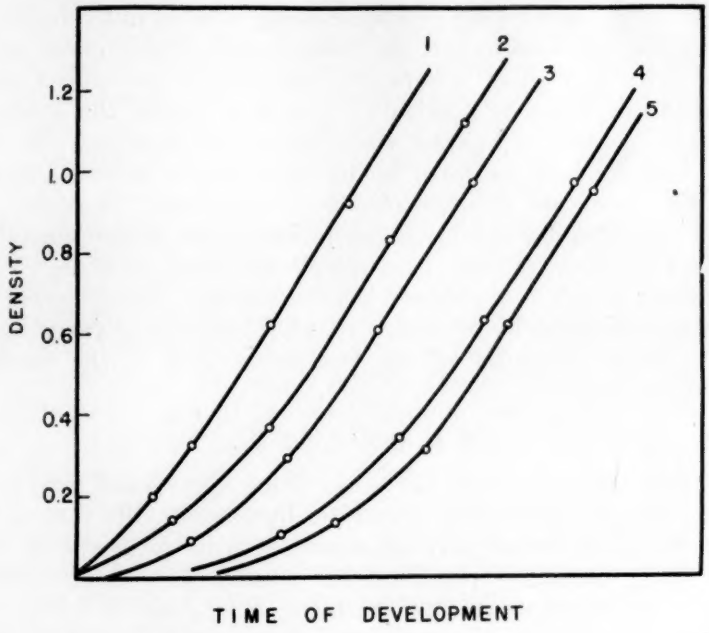

FIG. 2. Effect of bromide ion on development by ferro-oxalate. Motion-picture positive film "C" employed. pH = 4.76; log E = 1.15. Curve 1, no KBr, 0.0002 M phenosafranin added; Curve 2, no KBr or dye; Curve 3, 0.000167 M KBr; Curve 4, 0.000667 M KBr; Curve 5, 0.00167 M KBr.

ions. In such a case, development will show an induction period, the duration depending upon the length of time elapsing before the second phase of development can set in. This explanation of the induction period seems untenable, however. It gives no explanation of the marked dependence of the induction period upon the charge of the developer and upon the magnitude of the bromide barrier. Moreover, it is not at all clear why the induction period for a given developing agent may be varied from a fraction of a minute to several hours, simply by changing the concentration of the active ion. The rate of formation of interstitial silver ions is a function of the temperature of

the silver halide crystal, but not of the concentration of the developing agent.

It might be supposed that the observed increase in rate of density formation is simply a manifestation of the increase in the amount of silver-silver halide interface during the early stages of development. Actually, the increase in interface is accompanied by an increase in the rate of silver formation, but this increase in rate is not directly apparent from the density measurements. For example, when the uncharged agent diaminodurene is used as developer under conditions such that the majority of the developing grains are being reduced in a substantially parallel fashion (see subsequent discussion), the density curve has the form of Curve 0 in Fig. 1. There is no increase in the rate of density formation as development proceeds. When, however, the amount of reduced *silver* is plotted against the time of development, the curve exhibits a definite concave form in the early stages, showing that the *rate of silver formation is increasing* as development proceeds. The reason for this behavior lies in the variation of the silver-density ratio (Ag/D) with the average size of the developed silver particles.

Eggert and Küster have shown experimentally,[5] in agreement with a theoretical derivation by Nutting,[6] that the ratio Ag/D varies linearly with the average diameter of the projective area of the developed silver particles, i.e.,

$$\mathrm{Ag}/D = B \cdot \delta + C, \qquad (1)$$

where B and C are constants, the latter being quite small, and δ is the diameter. On the basis of the catalytic hypothesis, the rate of silver formation in development may be assumed to be proportional to the available silver interface, provided the equilibrium concentration of reactants is maintained at the interface. The rate of formation of silver at a given development center would then be approximately proportional to the apparent diameter of the silver area already formed, i.e.,

$$d\mathrm{Ag}/dt = k \cdot \delta. \qquad (2)$$

Rabinovitch and his co-workers [7] have published experimental results in agreement with this relation.

Differentiating eq. (1) with respect to t, and substituting in eq. (2), we obtain:

$$dD/dt = k'. \qquad (3)$$

Thus, under conditions where eqs. (1) and (2) hold good, the rate of formation of density is constant, even though the rate of formation of

[5] J. Eggert and A. Küster, *Kinotechnik*, **23**, 381 (1936).

[6] P. G. Nutting, *Phil. Mag.*, **26**, 423 (1913).

[7] A. J. Rabinovitch, A. N. Bogoyavlenski, and Y. S. Zuev, *Acta Physicochimica*, U.S.S.R., **16**, 307 (1942).

silver is increasing in the way expected from the autocatalytic nature of the reaction. In normal development, eq. (2) probably holds good only to an approximation and then only in the early stages of development when the interface can still be considered as growing freely. During the later stages of development, the interface will not increase as rapidly as indicated by eq. (1), and the rate of density formation will decrease.

The basic cause of the induction period apparently lies in a change which occurs in the magnitude of the bromide charge barrier as development proceeds.[1] The barrier existing in the vicinity of the latent-image nucleus may or may not be smaller than that which existed before exposure, but in either event, as development proceeds, the size of the silver speck increases and the magnitude of the barrier *in that localized region* drops. A larger percentage of the developer ions can then penetrate to the grain surface in the vicinity of the growing speck, and the effective concentration of the developing agent *increases* in that region. As a result, the development rate increases. The *relative* increase produced by a given decrease in barrier charge will be greater, the greater the charge of the developer ion. There will be no increase from this effect if the developer is an uncharged molecule.

The preceding considerations give the basis for an induction period in an individual grain. Some further considerations are required to account for the shapes of the density-time curves, such as those given in Fig. 1. In general, as development proceeds, the measured density might increase in either or both of two essentially distinct ways. An increase in density can be produced by increasing the *size* of the developed silver particles, i.e., by increasing the degree of completion of development of those grains which are developing. An increase in density can likewise be produced by essentially increasing the *number* of developing or developed grains.

Two extreme cases may be imagined. In the first, all of the grains start to develop at the same time, all have identical induction periods, and development of any one grain parallels that of any other grain. The increase in density then is caused solely by an increase in the size of the developing grains. In the second case, the induction periods vary in magnitude, and the duration of the induction period for any given grain is much greater than the duration of reaction beyond the induction period. At any intermediate stage of gross development of the emulsion, then, the image is made up essentially of completely or nearly completely developed grains. Other grains exist which will eventually develop (in a strict sense, they have already begun to develop, since reduction is taking place during the induction period), but at that moment they do not contribute measurably to the density. The density then is determined by the number of *developed* grains. In between these two extremes, the entire range of combinations of these

two factors is possible, with the increase in density resulting from an increase both in the number and in the average size of the developing grains.

The second extreme case described is often accepted as typical of normal development. Bagdasar'yan,[8] in a recent discussion of the induction period, states: "The duration of the induction period (for the individual grain) considerably exceeds the time of visible development. For example, the duration of visible development according to Meidinger was only about one minute, although the time of development of the emulsion was continued for fifteen to thirty minutes. It follows that the duration of the induction period under these conditions varied between several minutes and thirty minutes. With this relation of the durations of the induction period and of visible development, the kinetics of development of the emulsion must be determined by the law of the distribution of the induction periods. James's relation indicates that the valency of the developing ion has a definite effect on this law of the distribution." However, the emulsion which Meidinger used[9] was designed primarily to facilitate the microscopic study of the development of the individual grains. It was in no sense typical of the normal photographic emulsion. The emulsion was prepared from gelatin which had been inactivated by oxidation with iodine.* A large excess of potassium bromide was used, and a very thin coating from dilute gelatin solution was made on the microscope slide. The resulting emulsion had large, but very insensitive, grains. The large excess of bromide in the emulsion, together with the large amount of bromide which Meidinger used in the developer, would greatly accentuate the induction period, and the special gelatin and coating conditions might well lead to a relatively exceptionally high rate of reduction of the grain beyond the induction period. Rabinovitch and his co-workers,[7] whose results are also cited by Bagdasar'yan, used emulsions of essentially the same preparation.

There is evidence to show that, *under some conditions*, the increase in density at constant exposure during the development of a normal photographic emulsion is primarily a matter of increase in the size of the developing grains. These conditions were achieved in the following set of experiments. A simple positive-type film (which will be designated as "G") of relatively uniform grain size (Fig. 3) was selected. A pure hydroquinone solution buffered at pH = 8.8 and a ferro-oxalate solution of pH = 4.8 were used as developers. The film was given an

[8] Kh. S. Bagdasar'yan, *J. Phys. Chem.*, U.S.S.R., **17**, 336 (1943).

[9] W. Meidinger, *Physik. Z.*, **36**, 312 (1935); *Phot. Ind.*, **34**, 1305 (1936).

* S. E. Sheppard has pointed out (private communication) that the use of iodine in this process is liable to introduce a serious complication, because of the difficulty of removing iodine, or reaction products containing iodide, from the gelatin before subsequent use. The retention of iodine or iodide may be quite high.

exposure corresponding to a point well on the shoulder of the normal characteristic curve.

Figure 4 shows typical density-time curves obtained under the prescribed conditions. (The differences in the shape of these curves may be attributed to the action of the oxidation product of the hydroquinone.) The concave portion of each curve ends at a density of

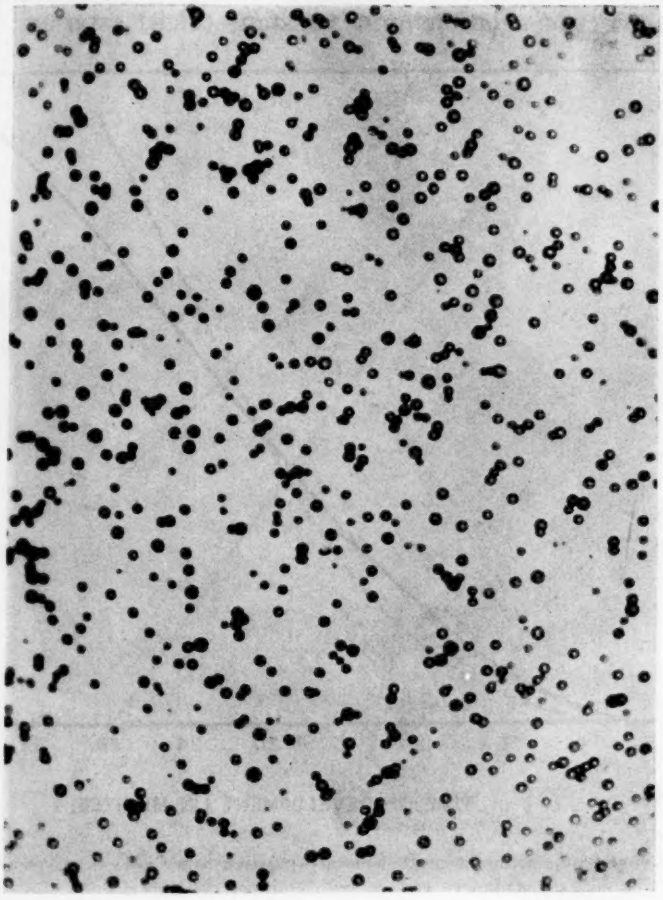

FIG. 3. Photomicrograph of undeveloped grains of film "G."

about 0.4. This is approximately 0.10 of the density obtained by maximum development. The percentage of actual silver formed is even less, however, and on this basis the measured gross "induction period" persists over only about five per cent. of the total course of development.

It can be shown in a number of ways that the increase of density with time of development is caused primarily by an increase in grain

size. One very useful indication is given by the determination of the silver-density relationship.[10] Figure 5 gives a logarithmic plot of silver against density for emulsion "G" after development in the ferro-oxalate solution. Curve 1 shows the relation for development carried nearly to completion and for variations in density produced by *varying the exposure*. In this case, density is varied effectively by changing the *number* of developed grains. The curve is a straight line of slope 0.96. Curve 2 shows the relation for a constant (high) exposure, and for

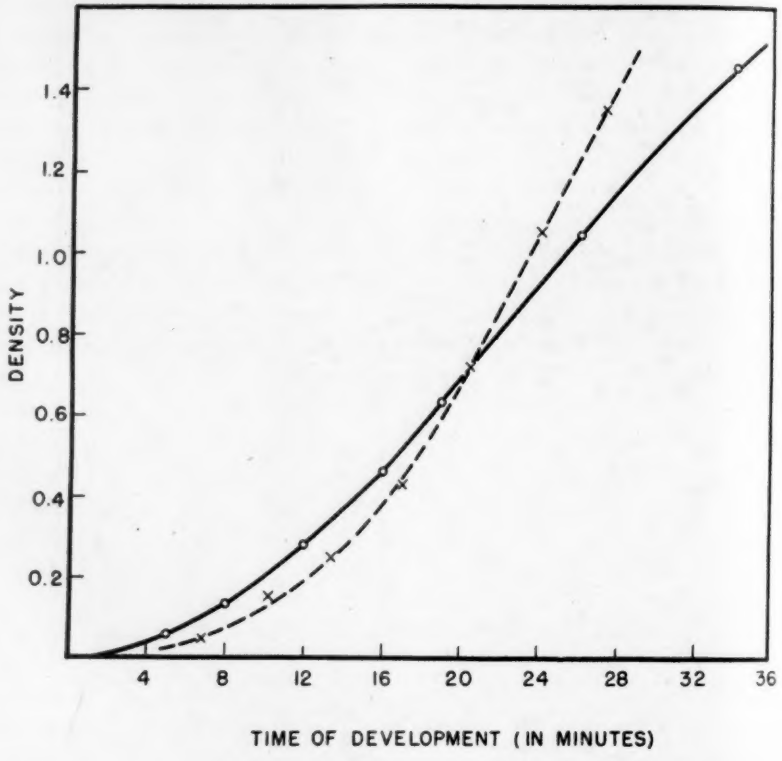

FIG. 4. Development curves for motion-picture positive film "G." ○ ○ = Ferro-oxalate developer; × × = hydroquinone developer.

different densities obtained by varying the time of development. Once again, a straight line is obtained, but this time the slope is approximately 1.43. This latter result is approximately that which would be expected on the basis of the Nutting-Eggert-Küster relationship, if the number of developing grains remained substantially constant and the size increased with increasing development time.[11] The results obtained

[10] S. E. Sheppard and A. Ballard, *J. Franklin Inst.*, **206**, 659 (1928).

[11] T. H. James, *J. Phys. Chem.*, **44**, 42 (1940).

with the hydroquinone developer were quite similar. (In the low exposure region, however, data have been obtained which indicate an important increase in number as well as size.)

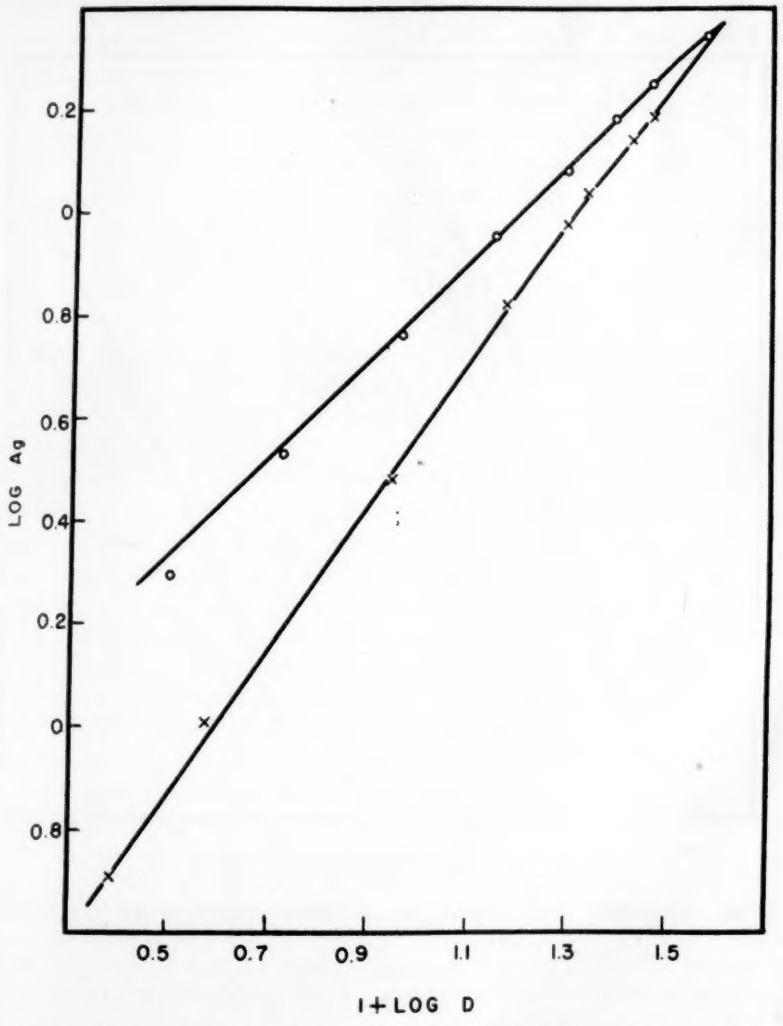

FIG. 5. Silver-density relationship for motion-picture positive film "G" developed in ferro-oxalate solution. ○-○ = Exposure varied; × × = development time varied.

The sensitivity of the induction period to bromide ion, in the case of the hydroquinone developer, furnishes a second method of investigating the problem of size *vs.* number. Figure 6 gives some typical curves for the present emulsion. Curve 1 represents the data obtained

when development was carried out in the absence of added bromide. Addition of potassium bromide to make a 0.001667 *M* solution resulted in a large increase in the concave toe region, with only a relatively small change in the development rate beyond the toe, as shown by Curve 2. When, however, development was started in a solution

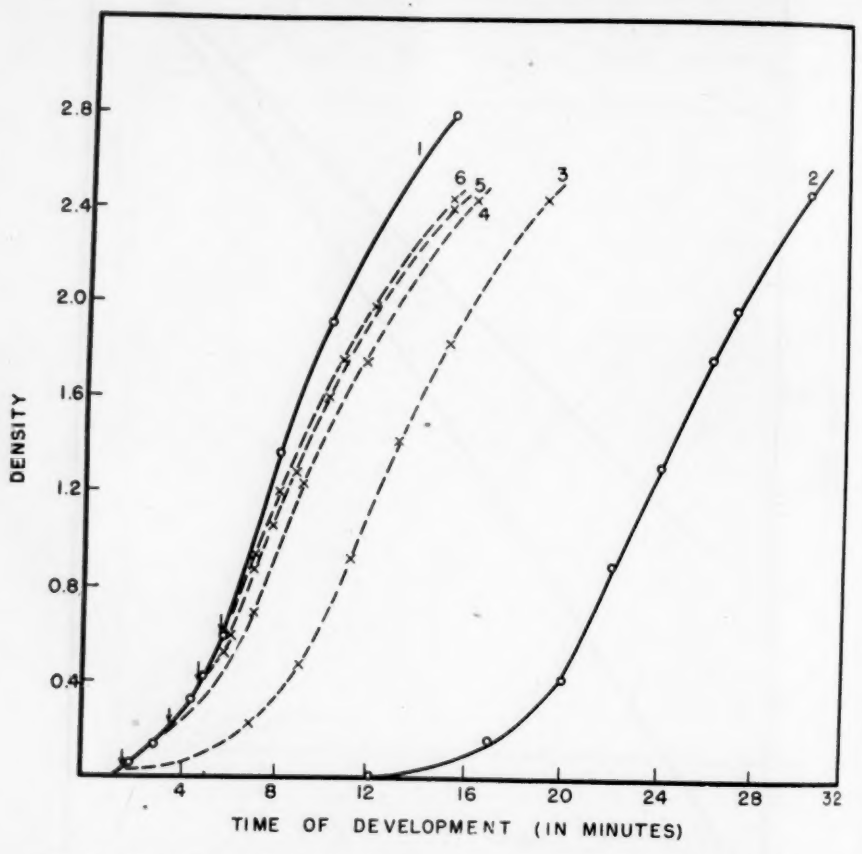

FIG. 6. Dependence of the bromide effect in development upon the point of addition of bromide to the developing solution. Pure hydroquinone developer of pH = 8.8. Motion-picture positive film "G." Curve 1, no bromide added; Curve 2, 0.001667 *M* KBr added before film was immersed in developer; Curves 3–6, KBr added after development had proceeded to points indicated by arrows.

containing no bromide, and the bromide was added after a density of only 0.03 was reached, development continued over the course indicated by Curve 3. When development was carried to a density of 0.20 in the bromide-free solution, and the addition was made at that point, continued development followed the course of Curve 4. Curves 5 and 6 represent similar experiments, the points of addition being indicated

by the arrows. In every case, the maximum slope of the development curve is the same as that of Curve 2, within the limits of experimental error. No essential difference was observed when, following the initial development in the bromide-free solution, the film was thoroughly washed and then immersed in a fresh hydroquinone solution containing 0.001667 *M* bromide.

Development by the ferro-oxalate developer is less sensitive to bromide in the early stages, but the results obtained in a set of experiments with this agent similar to those just described for hydroquinone, were otherwise in complete accord. Moreover, quite similar results were obtained when development was started in a ferro-oxalate solution, and comparison was then made of the continuation of development in hydroquinone solutions, one in the presence and the other in the absence of the potassium bromide.

These experiments show that by the time a density of only 0.03 is attained, development of the majority of the grains has proceeded to the point where the effect of bromide is greatly reduced. The majority of the grains have already "started to develop." By the time a density of 0.20 is reached, bromide no longer has a measurable selective effect upon the shape of the curve.

The general conclusions drawn from the photometric equivalent determinations and the bromide experiments are confirmed by microscopic examination of the developed and fixed emulsions. Such microscopic examinations do not give quantitative data, but the qualitative indications are quite definite. A series of such photomicrographs, made in these Laboratories, shows a progressive increase in average particle size as development in the pure hydroquinone solution was increased from three minutes to eight minutes. The film for three minutes' development (density of 0.14) showed a large number of silver particles, the great majority of which were considerably smaller than any of the grains shown in the photomicrograph of the undeveloped emulsion. The photomicrograph of the film developed five minutes (density of 0.43) showed no significant increase in number, but the average particle size was quite definitely greater. At eight minutes' development (density of 1.36), many individual particles were still distinguishable, and these averaged considerably larger than the particles at five minutes. Satisfactory micrographs could not be made at higher densities. In addition to the evidence of the photomicrographs, the change in grain size is indicated by the fact that, at very low densities, the image has a definite reddish-brown color, which disappears on further development.

The present data show quite definitely that, *under the conditions specified*, the increase in the *size* of the developing grains is the major factor in defining the general shape of the development curve. The data probably are not sufficiently precise to show that a distribution of

individual induction periods is not of importance in the very early stages of development. Bagdasar'yan's assumption [8] that the charge effect can contribute to an induction period only while development is in the submicroscopic state, however, is not necessarily valid. If the development of an individual grain started at only one center and proceeded smoothly from that point until the entire grain was reduced, the assumption should be valid. However, several centers of varying effectiveness may be involved over the course of development of the entire individual grain [12] (at least for the higher exposures), and new centers may even be formed during the course of development.[13] Thus, a manifestation of the barrier effect may still be found even after development of the grain has proceeded to a considerable extent.

In the region of lower exposures, it is no longer permissible to consider the gross induction period as a reliable measure of the induction period of an individual grain. Considerable variation in individual induction periods evidently occurs, and increase in the number as well as in the size of the developed silver particles becomes of importance in determining the increase in density. This is evident from numerous photomicrographic observations, such as those of Sheppard and Trivelli,[12] and is borne out by preliminary determinations of the silver-density relationship.

Factors other than the charge effect may, however, contribute to the measured form of the toe of the development curve. The time required for the developer to diffuse through the gelatin layer before it reaches the vicinity of the grain is short but finite. Experiments carried out by Burton in the Kodak Research Laboratories at Harrow have shown [14] that the apparent induction period in the development of the grains near the support side of the emulsion is greater than that of the grains near the outer side. The difference, however, amounts to half a minute or less for ordinary emulsion thicknesses. A second factor is introduced by the possibility that the small latent-image centers are less efficient catalysts for development, even when considered in terms of unit interface, than are the large centers or the silver of the partially developed grain.[8] This may be the explanation of the observation that zero-charged developing agents, under some conditions, yield short induction periods when the exposure is low. Such evidence is not indisputable, however, because the various grains under those conditions probably are not developing in a strictly parallel fashion. Moreover, eq. (1) does not hold for very small silver particles.

Under some practical conditions, the charge-factor tending to produce a concave toe in the development curve may be partially or

[12] Cf. S. E. Sheppard and A. P. H. Trivelli, *Proc. 7th Intern. Congress of Photography* (London), p. 174 (1928).

[13] T. H. James, *J. Chem. Physics*, 11, 338 (1943).

[14] P. C. Burton, Private communication.

completely masked by other effects. Even with a doubly charged developer, such as hydroquinone or glycin, the toe may practically disappear when the concentration and pH are high enough.[14] The effect of changing concentration at high pH values is illustrated for glycin in Fig. 7. The developer contained no sulfite or added bromide, and the pH value was adjusted to approximately 11.0 by the addition of 100 grams of sodium carbonate and x grams of potassium hydroxide per liter of solution. At the lower glycin concentrations, the toe is

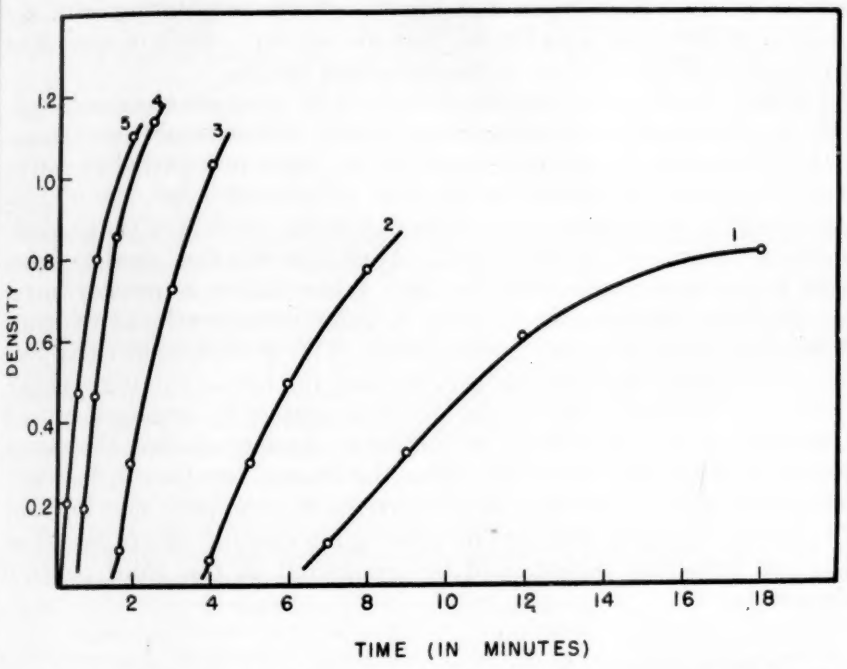

FIG. 7. Development by glycin at pH = 11, showing dependence of shape of curve upon concentration of developing agent. Motion-picture positive film "C" used. Log E = 0.55. Curve 1, 0.15 g. glycin per liter; Curve 2, 0.375 g./l.; Curve 3, 1.50 g./l.; Curve 4, 6.00 g./l.; Curve 5, 24.0 g./l.

quite marked, and the rate of development beyond the toe region varies with the concentration. In the region between 6 and 24 grams per liter, however, the rate of development beyond the toe region has become *independent of the concentration*, although the rate of development in the toe region still increases with the concentration. Hence, the toe becomes relatively smaller as the concentration increases, and practically vanishes at 24 grams per liter. This result, however, in no way invalidates our previous conclusions, which were based on comparisons of developers at equal concentration. Moreover, no induction

period was obtained with zero-charged developers even at great dilution, except for very low exposures.

The relative induction period decreases with increasing exposure and with decreasing bromide concentration in the developer. It also depends upon the nature of the emulsion. Considerably shorter induction periods have been obtained with high-speed, optically sensitized emulsions than with the simple, unsensitized materials of the type employed in the present work. Moreover, experiments with pinacyanol indicate that this typical sensitizing dye behaves much like phenosafranin in its action upon the kinetic effects originating with the repulsion of developer ions by the bromide barrier. Both of these dyes decrease the relative extent of the induction period.

Finally, the shapes of the two curves in Fig. 4 require some comment. Both developers act as doubly charged ions, and it would be expected on this basis that the relative extent of the induction period should be about the same. However, in the case of hydroquinone, the quinone formed during development (or a decomposition product of the quinone) increases the rate of development. Thus, the reaction product introduces a new autoaccelerating factor. When sulfite is present in the hydroquinone solution, the quinone is quickly removed and the autoacceleration from this source prevented. The development curve then assumes the same form as that representing the ferro-oxalate developer.

The accelerating action of the reaction product in development with hydroquinone in the absence of sulfite is associated with the charge effect. At high pH, its action upon the charge barrier can lead to a remarkable increase in contrast obtained upon incomplete development. This action, together with certain other photographic effects associated with the induction period, will be considered in the third paper of this series.

ROCHESTER, N. Y.,
March 29, 1945.

TEMPERATURE DISTRIBUTION IN TOROIDAL ELECTRICAL COILS OF RECTANGULAR CROSS SECTION.

BY

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ABSTRACT.

The essential content of a recent paper by the present writer comprises a comprehensive discussion of the physical bases underlying derivation of formulas for calculating the temperature distribution T, maximum temperature T_m and average temperature T_a in a toroidal electrical coil of rectangular cross section, internally generated heat and change of wire resistance with temperature being taken into account. Illustratively, the solution for the boundary value condition of constant surface temperature and uniform equivalent thermal conductivity was obtained.

For the most part, however, problems that arise in practice are not encompassed in the comparatively simple boundary conditions of constant temperature. Experiment shows that in general the boundary condition is $T - T' = -K\partial T/\partial n$; whereof n denotes the outward drawn normal to the coil surface, $K = (k_n/h)$ the ratio of the equivalent thermal conductivity in the direction of n to the emissivity of the boundary surface, and T and T' are the corresponding temperatures in the coil surface and the immediately adjacent ambient medium. Again, it frequently ensues in practice that the thermal conductivity is substantially different in the directions of the two principal axes of the cross section.

In the present paper formulas for T, T_m and T_a are obtained for electrical coils of ratio of external to internal radius greater than (roughly) two whereof (i) the thermal conductivity is different in the directions of the two principal axes of the cross section, (ii) K is different on but constant over each of the four faces of the coil, and (iii) no restriction is made as to T' except that over each face it be expressible in a generalized Fourier series. Determination of T is posed as a boundary-value problem in the mathematical theory of heat; the formal solution of T effected by expansions in orthogonal functions; and T_m and T_a then determined through use of their known relationships with T. The resulting formulas are in the form of rapidly-converging singly-infinite trigonometric-hyperbolic series. Illustrative of application of these general formulas, the maximum temperatures in a coil of given dimensions subject to two different sets of surface conditions are calculated and found to be in excellent agreement with the known measured values.

The just-mentioned formulas encompass practically all cases encountered in practice except those coils which do not satisfy the restriction as to ratio of radii. For these latter formulas for T, T_m and T_a are obtained pursuant to conditions of (i) equivalent thermal conductivity different in the directions of the two principal axes of the cross section, (ii) K, and likewise T', different on but constant over each of the four faces of the coil. These formulas are in the form of rapidly-converging singly-infinite trigonometric-Bessel function (of zero order) series. Illustratively, the maximum temperature in a coil of given dimensions is calculated and found to be in excellent agreement with the known measured value.

1. GENERAL REMARKS.

A current-carrying toroidal coil of rectangular cross section, comprised of many turns of insulated wire of circular or rectangular cross section, is an important component of many electrical devices. In use

the heat generated in the coil results in a nonuniform temperature rise throughout the coil. In consequence, a point of maximum temperature is to be found in the coil. Now in that the permissible maximum coil current is that which produces a specified maximum temperature in the coil; and in that the "hot" or "operating" resistance of the coil depends on, and can be calculated from the average (mean) temperature of the coil, formulas for predetermining the temperature distribution T, maximum temperature T_m, and average temperature T_a are of considerable interest to the designer of toroidal coils.

A comprehensive discussion of the physical bases underlying derivation of such formulas comprises the essential content of a recent lengthy paper by the writer.[1] For explicit details the reader is referred to this paper. For present purposes it suffices to remark that following epitomization of the work of Binder,[2] Humburg,[3] Jakob,[4,5] Ott,[6] Punga,[7] Rogowski[8] and Vidmar[9] a detailed consideration of the individual and collective effects of the various physical and geometric factors involved leads to the conclusion that if a toroidal coil (i) is comprised of many turns of thinly insulated wire of comparatively small cross section, (ii) is of rectangular cross section, (iii) is of a ratio of outer to inner radius greater than (roughly) two, and (iv) if—as is usually the case—the temperature distribution is the same over all radial cross sections of the coil, the temperature distribution in the coil approximates that in an infinite "cylindrical" coil of the same structure, cross section, and surface conditions; whence determination of T, T_m, and T_a for the coil—internally generated heat and change of wire resistance with temperature being taken into account—can be posed as a two-dimensional (rectangular) boundary-value problem in the mathematical theory of heat, capable of solution in Cartesian coordinates by known methods. Illustratively, the solution for the boundary value condition of constant surface temperature is obtained. It is shown that this solution encompasses, as special cases, the earlier analytic solution of Humburg,[3] Jakob,[4,5] and Rogowski.[8]

For the most part, however, problems that arise in practice are not encompassed in the comparatively simple boundary condition of constant temperature. Experiment shows that in general the boundary condition is $T - T_i = -K\partial T/\partial n$: whereof, n denotes the outward drawn normal to the coil surface; $K = k_n/h$, the ratio of the equivalent

[1] T. J. Higgins, *Trans. A. S. M. E.*, **66**, 665–670 (1944).
[2] L. Binder, *Arch. f. Elek.*, **2**, 131–148 (1913).
[3] K. Humburg, *Elec. u. Masch.*, **27**, 677–681, 696–708 (1909).
[4] M. Jakob, *Arch. f. Elek.*, **8**, 117–126 (1919).
[5] M. Jakob, *Trans. A. S. M. E.*, **65**, 593–602 (1943).
[6] L. Ott, *Mitt. ü. Forsch. a. d. Geb. d. Ing.*, Heft. 35 and 36, 53–107 (1906).
[7] F. Punga, *Arch. f. Elek.*, **21**, 97–105 (1928).
[8] W. Rogowski, *Arch. f. Elek.*, **7**, 41–47 (1918–1919).
[9] M. Widmar, *Elek. u. Masch.*, **36**, 49–52, 64–66 (1918).

thermal conductivity in the direction of n to the emissivity of the boundary surface; and T and T_i are the corresponding temperatures in the coil surface and the immediately adjacent ambient medium. Again, the solutions [1,3,4,5,8] mentioned above are for equivalent thermal conductivity k uniform throughout the medium; hence are not applicable to coils whereof k is substantially different in the directions of the two principal axes of the cross section.

Now solutions encompassing both different "principal" equivalent thermal conductivities and the general boundary condition have been advanced by Roth [10] and by Buchholz [11]—*but only for special cases of the general problem.* Thus Buchholz, though taking into account variation of wire resistance with temperature, constrains himself to the case of T_i identical over all four sides and K different on adjacent sides but the same on opposite sides of the rectangular cross section. On the other hand, though Roth encompasses somewhat less limited boundary conditions—values of T_i, and likewise K, constant but different on each of the four sides—he has not, because—as he remarks—of the analytical difficulties thereby introduced, taken into account the change of wire resistance with temperature. In consequence, though the formulas of Roth and Buchholz encompass many of the situations encountered in practice, others of much practical importance are not covered: for example, that of calculating the temperature in a rectangular toroidal coil wound in the slot of a channeled spool, wound on a heat-conducting metal core, or otherwise so disposed that the boundary conditions are such that they are not encompassed in Buchholz's solution and whereof the electrical loading is such that the effect of change of wire resistance with temperature, neglected in Roth's solution, is of consequence (the considerable importance of even small variations of the wire resistivity over the cross section is strikingly illustrated by the curves contained in Jakob's [5] paper).

Formulas for such a case—and for a host of others—are, however, afforded by the solution (in Section 2) of the following very general problem in the mathematical theory of heat:

I. The axis of a homogeneous infinite rectangular cylinder coincides with the z axis of a system of rectangular coordinates; the sides of the rectangular cross section S of width $2a$ and height $2b$ are directed parallel to the x and y axes. The thermal conductivities in the x and y directions are k_x and k_y. The rate of internally-generated heat density is $q_0(1 + \alpha_0 T)$ whereof q and α_0 are arbitrary constants. From the face of the cylinder f_i $(i = 1, 2, 3, 4)$ heat is transferred to the adjacent medium according to the relationship $T - T_i = -K_i \partial T/\partial n$. The K_i are different on but constant over each f_i. No restriction is

[10] E. Roth, *Bull. Soc. Fran. d. Elec.*, [2] 7, 840–966 (1927).
[11] H. Buchholz, *Arch. f. Elek.*, 28, 122–130 (1934).

made as to the distribution of T_i over f_i except that it be expressible in a generalized Fourier series. Required: the temperature distribution $T(x, y)$, the maximum temperature T_m, and the average temperature of the cylinder T_a.

Formulas for these temperatures obtained, to apply them to calculation of coil temperatures we need only (as is detailed in reference 1) interpret k_x and k_y as the principal thermal conductivities of the coil (analytic methods of calculating these constants and physical data essential thereto are to be found in references 5 and 10); α_0 as the temperature coefficient of resistance at 0° C. of the coil wire; $q_0 = I^2R_0/V$ as the general heat power density at 0° C., I the coil current, R_0 the coil resistance at 0° C. and V the coil volume; select values of K_i appropriate to the given surface conditions (numerical values for various materials and surface conditions are to be found in reference 10); express the temperature distribution in the ambient medium adjacent to each of the four faces in terms of equivalent generalized Fourier series; and through appropriate substitution of these data obtain T, T_m, and T_a for the coil.

The formulas that result from the solution of I encompass as special cases the formulas of Buchholz and Roth. It may be noted here, however, that the corresponding forms of our general formulas are much more convenient to numerical computation than are those of Buchholz and Roth. The latter's, for instance, are couched in double Fourier series which converge much more slowly than do the singly-infinite series of our formulas.

Broad, however, as is the class of problems encompassed by the formulas stemming from I, it does not embrace coils of internal radius such that determination of the temperature distribution over the cross section necessitates consideration of the actual toroidal geometry. But in that such coils are utilized in practice it is obviously desirable to have apposite formulas available for design. To the writer's knowledge, no such formulas are to be found in the literature. These desired formulas, however, are afforded by the solution (in Section 5) of the following very general problem in the mathematical theory of heat:

II. The axis of a homogeneous toroid of rectangular cross section S, external radius a, and internal radius a' coincides with the z axis of a system of cylindrical coordinates; the sides of S, of width $a - a'$ and height $2b$, are directed parallel to the r and z axes. The thermal conductivities in the r and z directions are k_r and k_z. The rate of internally-generated heat density is $q_0(1 + \alpha_0 T)$. From the face of the toroid f_i $(i = 1, 2, 3, 4)$ heat is transferred to the adjacent medium T_i according to the relationship $T - T_s = -K_i \partial T/\partial n$. The K_i are different on but constant over each f_i. Required: the temperature distribution $T(r, z)$, the maximum temperature T_m, and the average temperature of the toroid.

Formulas for these temperatures obtained, application to electrical coils is in much the same fashion as is described *re I*. It may be noted that though the boundary conditions are less general than those specified for *I*: (i) they encompass most of the problems of practice, and (ii) if desired, formulas for the more general conditions corresponding to those of *I* can be easily obtained by paralleling the solution of *I*.

Details apposite to numerical computation with the general formulas stemming from the solutions of *I* and *II* are illustrated by the numerical examples of Sections 4 and 7 wherein maximum temperatures in coils of given dimensions subjected to special boundary conditions are calculated and found to be in excellent agreement with corresponding values obtained by experiment.

2. SOLUTION OF PROBLEM I.

From the mathematical theory of heat we have to find $T(x, y)$ that satisfies the partial differential equation

$$k_x\partial^2T/\partial x^2 + k_y\partial^2T/\partial y^2 = -q_0(1 + \alpha_0T) \tag{1}$$

over S, and the boundary conditions

$$T - T_1 = -K_1\partial T/\partial x, \qquad x = a, \tag{2}$$

$$T - T_2 = K_2\partial T/\partial x, \qquad x = -a, \tag{3}$$

$$T - T_3 = -K_3\partial T/\partial y, \qquad y = b, \tag{4}$$

$$T - T_4 = K_4\partial T/\partial y, \qquad y = -b. \tag{5}$$

Rearranging (1) yields

$$\partial^2T/\partial x^2 + k\partial^2T/\partial y^2 + C^2T = -C', \tag{6}$$

wherein

$$k = k_y/k_x; \qquad C^2 = q_0\alpha_0/k_x; \qquad C' = q_0/k_x. \tag{7}$$

Rearranging (2) to (5) yields

$$T + K_1\partial T/\partial x = \sum_{i=1}^{\infty} S_i \cos(s_iy + s_i'), \qquad x = a, \tag{8}$$

$$T - K_2\partial T/\partial x = \sum_{i=1}^{\infty} S_i' \cos(s_iy + s_i'), \qquad x = -a, \tag{9}$$

$$T + K_3\partial T/\partial y = \sum_{i=1}^{\infty} P_i \cos(p_ix + p_i'), \qquad y = b, \tag{10}$$

$$T - K_4\partial T/\partial y = \sum_{i=1}^{\infty} P_i' \cos(p_ix + p_i'), \qquad y = -b, \tag{11}$$

wherein the right-hand members are respectively the generalized

Fourier expansions of T_1 and T_2 over $-b \leq y \leq b$ and of T_3 and T_4 over $-a \leq x \leq a$.

The procedure for solving (6) pursuant to (8) to (11) is: symbolizing the left-hand member of (6) as an operator $\nabla = (\partial^2/\partial x^2 + k\partial^2/\partial y^2 + C^2)$ operating on T, we find a particular solution of (6); add to this functions F involving arbitrary constants so related that $\nabla F = 0$; whence the resulting sum is also a solution of (6); substitute this resulting expression in (8) to (11); equate coefficients of corresponding sine and cosine terms of the like infinite series in the left and right-hand members of each equation; solve the set of equations so obtained for values of the constants; whence T is determined. In this fashion we obtain—the lengthy, purely formal details are to be found in the writer's [12] recent paper on temperature determination in transformer cores, whereof it is necessary to solve (6) pursuant to (8) to (11)—

$$T = \sum_{i=1}^{\infty} A_i \cos(m_i x + m_i') \cosh(n_i y + n_i')$$

$$+ \sum_{i=1}^{\infty} B_i \cos(p_i x + p_i') \cosh(q_i y + q_i')$$

$$+ \sum_{i=1}^{\infty} C_i \cosh(r_i x + r_i') \cos(s_i y + s_i') + \sum_{i=1}^{\infty} D_i \cos(m_i x + m_i'), \quad (12)$$

whereof, given values of K_i ($i = 1, 2, 3, 4$) pertinent to a particular problem, we obtain the apposite values of m_i, m_i', p_i, p_i', s_i, and s_i' by solving the pairs of equations

$$\tan(m_i a + m_i') = 1/K_1 m_i; \qquad \tan(m_i a - m_i') = 1/K_2 m_i; \quad (13)$$

$$\tan(p_i a + p_i') = 1/K_1 p_i; \qquad \tan(p_i a - p_i') = 1/K_2 p_i; \quad (14)$$

$$\tan(s_i b + s_i') = 1/K_3 s_i; \qquad \tan(s_i b - s_i') = 1/K_4 s_i; \quad (15)$$

the remaining constants then being determined from

$$n_i = [(m_i^2 - C^2)/k]^{1/2}, \quad (16)$$

$$q_i = [(p_i^2 - C^2)/k]^{1/2}, \quad (17)$$

$$r_i = (k s_i^2 - C^2)^{1/2}, \quad (18)$$

$$n_i' = \tanh^{-1}(K_4 - K_3)[(2/n_i) + (K_3 + K_4)\coth n_i b]^{-1}, \quad (19)$$

$$q_i' = \tanh^{-1} \frac{[(\cosh q_i b + K_i q_i \sinh q_i b)P_i' - (\cosh q_i b + K_2 q_i \sinh q_i b)P_i]}{[(\sinh q_i b + K_1 q_i \cosh q_i b)P_i' - (\sinh q_i b + K_2 q_i \cosh q_i b)P_i]} \quad (20)$$

[12] T. J. Higgins, *Trans. A. I. E. E.*, **64**, 190–194 (1945).

$$r_i' = \tanh^{-1} \frac{[(\cosh r_i a + K_3 r_i \sinh r_i a) S_i' - (\cosh r_i a + K_4 r_i \sinh r_i a) S_i]}{[(\sinh r_i a + K_3 r_i \cosh r_i a) S_i' - (\sinh r_i a + K_4 r_i \cosh r_i a) S_i]}, \tag{21}$$

$$A_i = -D_i[\cosh(n_i b + n_i') + K_3 n_i \sinh(n_i b + n_i')]^{-1}, \tag{22}$$

$$B_i = P_i[\cosh(q_i b + q_i') + K_3 q_i \sinh(q_i b + q_i')]^{-1}, \tag{23}$$

$$C_i = S_i[\cosh(r_i a + r_i') + K_1 r_i \sinh(r_i a + r_i')]^{-1}, \tag{24}$$

$$D_i = (4C' \sin m_i a \cos m_i')(m_i^2 - C^2)^{-1}(2m_i a + \sin 2m_i a \cos 2m_i')^{-1}. \tag{25}$$

The average temperature over the cross section is given by

$$T = (1/4ab) \int_{-a}^{a} \int_{-b}^{b} T dy dx. \tag{26}$$

Substituting (12) in (26) and integrating yields

$$T_a = (1/ab)[\sum_{i=1}^{\infty} A_i \sin m_i a \cos m_i' \sinh n_i b \cosh n_i'/m_i n_i$$

$$+ \sum_{i=1}^{\infty} B_i \sin p_i a \cos p_i' \sinh q_i b \cosh q_i'/p_i q_i$$

$$+ \sum_{i=1}^{\infty} C_i \sin s_i b \sin s_i' \sinh r_i a \cosh r_i'/r_i s_i$$

$$+ \sum_{i=1}^{\infty} D_i b \sin m_i a \cos m_i'/m_i]. \tag{27}$$

T is a function of two variables. The form of T, the sum of four infinite series, renders unfeasible determination of the maximum temperature by application of the rules of the calculus for determining maxima and minima of functions of two variables. In general, the maximum temperature is to be ascertained by plotting the temperature distribution over that region of the cross section which a preliminary calculation, or consideration of given physical conditions, indicates as encompassing T_m. If, however, the temperature distribution is symmetrical about each of the coordinates axes (i.e., $K_1 = K_2$, $K_3 = K_4$, $S_i = S_i'$, $P_i = P_i'$), T_m occurs at $x = 0$, $y = 0$. Hence, *for this case*, often encountered in practice,

$$T_m = \sum_{i=1}^{\infty} A_i \cos m_i' \cosh n_i' + \sum_{i=1}^{\infty} B_i \cos p_i' \cosh q_i'$$

$$+ \sum_{i=1}^{\infty} C_i \cosh r_i' \cos s_i' + \sum_{i=1}^{\infty} D_i \cos m_i'. \tag{28}$$

By virtue of the symmetry of (6) and of (8) to (11) alternative

formulas for T, T_a, and T_m can be obtained by interchanging x and y and corresponding constants (i.e., a and b, K_1 and K_3, m_i and n_i, etc.) in (12), (27), and (28). As this is easily done—illustratively, see Section 4—we shall not take space to write down these alternative forms.

3. SOME SIMPLIFIED FORMULAS.

In applying the formulas derived in the previous section to the calculation of temperatures in coils one or more sides of which are in contact with metal or are covered with a thin layer of insulation through which heat flows substantially normal to the interface between coil and insulation and is radiated to a surrounding ambient medium, a considerable simplification of calculation stems from the use of a theorem advanced by Fourier: through assignment of suitable values of K_i to the individual faces of the coil (these values determined by experience or by calculation with the formulas given in most reference texts on heat) the coil can be considered as radiating into a surrounding ambient medium whereof $T_i = T_s$ is identical over all faces of the coil. But if T_i is constant everywhere, (12), (27), and (28) of the preceding section can be so transformed that the new forms require but a tithe of the computation otherwise necessary.

Thus, if $T_1 = T_2 = T_3 = T_4 = T_s$, write

$$T = t + T_s; \qquad T_i = t_i + T_s. \tag{29}$$

Substituting (29) in (1) yields

$$\partial^2 t/\partial x^2 + k\partial^2 t/\partial y^2 + C^2 t = -c', \tag{30}$$

whereof

$$c' = q_0(1 + \alpha_0 T_s)/k_x, \tag{31}$$

and C^2 is given by (7). Substituting (29) in (2) to (5) yields

$$t - t_1 = -K_1 \partial t/\partial x, \qquad x = a, \tag{32}$$

$$t - t_2 = K_2 \partial t/\partial x, \qquad x = -a, \tag{33}$$

$$t - t_3 = -K_3 \partial t/\partial y, \qquad y = b, \tag{34}$$

$$t - t_4 = K_4 \partial t/\partial y, \qquad y = -b. \tag{35}$$

By virtue of the formal identity of (30) with (1) and of (32) to (35) with (2) to (5) we have at once that the solution for t is afforded by that for T if therein we replace C' by c' and interpret S_i, S_i', P_i and P_i' as the coefficients of the generalized Fourier expansions of, respectively, t_1, t_2, t_3, t_4. But, and this is the crux, as by (29) and our given condition ($T_1 = T_2 = T_3 = T_4 = T_s$) we have $t_1 = t_2 = t_3 = t_4 = 0$, these Fourier coefficients are each 0. Accordingly, determination of the constants in (12)—T now interpreted as t—is very considerably simplified. A similar argument is valid for t_m and t_a, defined by

$$T_m = t_m + T_s; \qquad T_a = t_a + T_s. \tag{36}$$

In conclusion we note that if all points of the coil are originally at ambient temperature, the lower case values are the temperature rises above the initial ambient value.

4. A NUMERICAL EXAMPLE

To illustrate use of the formulas derived in Sections 2 and 3 we consider the following problem: to calculate the temperature at a certain point in the rectangular toroidal coil of Run 9*B* of Rayner's[13] National Physics Laboratory experiments on dynamo field coils. We select this particular run as Roth also used the data thereof to check the accuracy of his formulas; whence our use of it not only provides the desired illustrative problem but also brings out how much superior *re* quickness and ease of numerical calculation are those special forms of our general formulas which are equivalent to Roth's formulas.

Given data: Shunt field coil of rectangular cross section: 2 in. × 10.75 in., wound in metal channel approximately 2.1 in. deep bent into rectangular cross section giving external coil dimensions of 19.0 in. × 17.5 in. Winding comprised of 550 turns of double cotton-covered copper wire, these turns arranged in 11 layers, each of 50 turns. Volume of coil 1500 cu. in. Resistance at 0° C. 0.688 ohms; hot resistance 0.818 ohms. Coil current 19 amperes. Room temperature 18.5° C., coil surface temperature (by thermometer) 40° C., maximum internal coil temperature 45.7° C. at ($x = -0.693$ cm., $y = 0$), temperature 37.5° C. at ($x = 2.08$ cm., $y = 0$). Outer 10.75 in. surface radiates to air; inner three surfaces in contact with metal of channel form. Assembly of coil and form mounted on field pole and data taken with machine running light.

Derived data: The per cent electrical conductivity of the copper wire is not stated. Taking it to be 100 per cent., we have (as can be found in any electrical handbook) that $\alpha_0 = 0.00423°\ \text{C.}^{-1}$. From given data $q_0 = I^2R_0/V = 0.0101$ watts cm.$^{-3}$ Roth cites (from tables in his paper comprised of data compiled by numerous investigators) as typical values: $k_x = k_y = 0.01045$ watt cm.$^{-1}$ °C.$^{-1}$; $h_1 = 0.00158$, $h_2 = 0.00083$, $h_3 = h_4 = 0.00214$ watt cm.$^{-2}$ °C.$^{-1}$; whence $K_1 = 6.62$, $K_2 = 12.6$, $K_3 = K_4 = 4.88$ cm. The values of h, hence K, are such that the analysis of Section 3 is applicable. Because of the layer of hot air immediately adjacent to the surface, $T_1 = T_2 = T_3 = T_4 = T_s$ will be higher than room temperature but substantially lower than the coil surface temperature; say $T_s = 27.5°$ C.

Formulas: Inasmuch as $K_3 = K_4$, but $K_1 \neq K_2$, use of the alternative formulas mentioned at the close of Section 2 will enable us to avoid the necessity of solving a pair of equations of the type of (13). Accordingly, noting that by virtue of the given condition all constants vanish except A_i, D_i, m_i, m_i' and n_i; that $y = 0$ for the two points

[13] E. H. Rayner, *J. I. E. E.*, **34**, 613–659 (1904–1905).

whereat the temperature is to be calculated; and interchanging x and y and corresponding constants, we obtain from (12) and (29).

$$T = T_s + \sum_{i=1}^{\infty} A_i \cosh (m_i x + m_i') + \sum_{i=1}^{\infty} D_i,$$

wherein from (22), (25), (19), (16), (13), and (31) we have

$$A_i = - D_i[\cosh (m_i a + m_i') + K_1 m_i \sinh (m_i a + m_i')]^{-1},$$

$$D_i = (4c' \sin n_i b)(n_i^2 - C^2)^{-1}(2n_i b + \sin 2n_i b)^{-1},$$

$$m_i' = \tanh^{-1} (K_2 - K_1)[(2/m_i) + (K_1 + K_2) \coth m_i a]^{-1},$$

$$m_i = (n_i^2 - C^2)^{1/2},$$

$$\tan n_i b = 1/K_3 n_i,$$

$$C^2 = q_0 \alpha_0 / k_x; \qquad c' = q_0(1 + \alpha_0 T_s)/k_x.$$

Computation: Substituting appropriately we obtain:

$C^2 = 0.00408$; $c' = 1.075$;

$n_1 = 0.0858$; $m_1 = 0.0575$; $m_1' = 0.082$; $D_1 = 392$; $A_1 = - 375$;

whence we obtain for the maximum temperature

$$T_m = 27.5 + 17.0 - 0.0^+ = 44.5^\circ \text{ C.}$$

Comparing this value with the measured value 45.7° C. we find a difference of 1.2°.

Again, for the point ($x = 2.08$, $y = 0$) we find

$$T = 27.5 + 9.2 - 0.0^+ = 36.7.$$

The corresponding measured value is 37.5° C., a value 0.8° C. larger.

Run 9*A* of Rayner's test is with the same coil covered with a half-inch thickness of "canvas, fullerboard, millboard and string." Computation omitted here yields a maximum temperature of 64.1° C. compared with the measured value of 65.4° C.

These three computed values are about 1° lower than the corresponding measured values. But in that our assumed value for T_s may be too low by 1° or so, or in that variations between the assumed and actual constants may introduce errors of a degree or so, we interpret the calculated and measured values as being in substantial agreement.

Comparison of our computations with the corresponding ones of Roth reveals the decided superiority of our formulas. Thus, use of our alternative formulas eliminated the necessity of solving a pair of equations of the form (13), not a simple task; Roth needs do so. Again, where three terms must be calculated in his doubly infinite series, we need calculate but one in our singly-infinite series to obtain the same degree of convergence. And yet other advantages are apparent.

5. SOLUTION OF PROBLEM II.

From the mathematical theory of heat we have to find $T(r, z)$ that satisfies the partial differential equation

$$k_r(\partial^2 T/\partial r^2 + r^{-1}\partial T/\partial r) + k_z\partial^2 T/\partial z^2 = -q_0(1 + \alpha_0 T) \tag{37}$$

over S, and the boundary conditions

$$T - T_s = -K_1\partial T/\partial r, \qquad r = a, \tag{38}$$

$$T - T_s = K_2\partial T/\partial r, \qquad r = a', \tag{39}$$

$$T - T_s = -K_3\partial T/\partial z, \qquad z = b, \tag{40}$$

$$T - T_s = K_4\partial T/\partial z, \qquad z = -b. \tag{41}$$

Substituting

$$T = t + T_s \tag{42}$$

in (37) yields

$$\partial^2 t/\partial r^2 + r^{-1}\partial T/\partial r + C^2 T + k\partial^2 t/\partial z^2 = -C', \tag{43}$$

wherein

$$k = k_z/k_r;\ C^2 = q_0\alpha_0/k_r;\ C' = q_0(1 + \alpha_0 T_s)/k_r;\ C^2/C' = T_s + \alpha_0^{-1}. \tag{44}$$

Substituting (42) in (38) to (41) yields

$$t = -K_1\partial t/\partial r, \qquad r = a, \tag{45}$$

$$t = K_2\partial t/\partial r, \qquad r = a', \tag{46}$$

$$t = -K_3\partial t/\partial z, \qquad z = b, \tag{47}$$

$$t = K_4\partial t/\partial z, \qquad z = b. \tag{48}$$

The procedure for solving (44) pursuant to (45) to (48) is effected in much the same fashion as in Section 2. Take

$$t = \sum_{i=1}^{\infty} [A_i J_0(m_i r) + B_i N_0(m_i r)] \cos(n_i z + n_i') + \sum_{i=1}^{\infty} D_i \cos(n_i z + n_i'), \tag{49}$$

wherein J_0 and N_0 designate Bessel functions of zero order of the first and second * kind. Substituting (49) in (43) yields

$$\sum_{i=1}^{\infty} D_i(C^2 - kn_i^2) \cos(nz + n_i') = -C', \tag{50}$$

providing

$$m_i^2 = C^2 - kn_i^2. \tag{51}$$

* There are several definitions of the Bessel function of the second kind. We use that tabulated in Jahnke and Emde's tables.

Substituting in (50) the generalized Fourier expansion of C' derived in the appendix and equating corresponding coefficients of the cosine terms yields

$$D_i = -4C' \sin n_i b \cos n_i'/m_i^2(2n_i b + \sin 2n_i b \cos 2n_i'). \quad (52)$$

Substituting (49) in (45), equating coefficients of corresponding cosine terms, and repeating for (46) yields two linear algebraic equations in A_i and B_i. Solving these we have

$$A_i = D_i\Delta^{-1}[N_0(m_i a) - N_0(m_i a')] - m_i[K_1 N_1(m_i a) + K_2 N_1(m_i a')], \quad (53)$$

and

$$B_i = -D_i\Delta^{-1}[J_0(m_i a) - J_0(m_i a')] - m_i[K_1 J_1(m_i a) + K_2 J_1(m_i a')], \quad (54)$$

wherein

$$\Delta = [J_0(m_i a) - K_1 m_i J_1(m_i a)][N_0(m_i a') + K_2 m_i N_1(m_i a')] - [J_0(m_i a') + K_2 m_i J_1(m_i a')][N_0(m_i a) - K_1 m_i N_1(m_i a)], \quad (55)$$

and J_1 and N_1 designate Bessel functions of the first order. Substituting (49) in (47) we have satisfaction of the resulting equation providing

$$\tan (n_i b + n_i') = 1/K_3 n_i. \quad (56)$$

Similarly, from (48) we obtain

$$\tan (n_i b - n_i') = 1/K_4 n_i. \quad (57)$$

For particular values of K_3 and K_4 this pair of equations yields the desired values of n_i and n_i'.

The average temperature over the cross section is given by

$$T = T_s + [1/2b(a - a')] \int_{a'}^{a} \int_{-b}^{b} t dz dr. \quad (58)$$

Substituting (49) in (58) and integrating yields

$$T_a = T_s + [1/b(a - a')][\sum_{i=1}^{\infty} 2(n_i m_i)^{-1}\{\sum_{j=0}^{\infty} A_i[J_{2j+1}(m_i a) - J_{2j+1}(m_i a')] + B_i[N_{2j+1}(m_i a) - N_{2j+1}(m_i a')]\} \sin n_i b \cos n_i' + \sum_{i=1}^{\infty} D_i(a - a') \sin n_i b \cos n_i'/n_i]. \quad (59)$$

As in Section 2, T is a function of two variables; whence the maximum temperature is to be found by plot of T over that region wherein the maximum is to be expected. In particular, if $K_1 = K_2$ and $K_3 = K_4$ the maximum temperature will occur in the plane $z = 0$ very close to

the center $(a_0, 0)$ of the cross section. An excellent approximation for this condition is, then, from (49)

$$T_m \cong T_s + \sum_{i=1}^{\infty} [A_i J_0(m_i a_0) + B_i N_0(m_i a_0)] \cos n_i' + \sum_{i=1}^{\infty} D_i \cos n_i'. \quad (60)$$

6. SOME SIMPLIFIED FORMULAS

If, as is occasionally the case, $K_1 = K_2 = K_3 = K_4 = 0$ and $k = 1$, (49) and (60) can be written in forms more convenient to numerical computation. For by (56) and (57) $n_i' = 0$ and $n_i b = (2i - 1)\pi/2$ and by (51) $m_i^2 = C^2 - n_i^2$ whence (52) becomes

$$D_i = -2(-1)^{i-1} C'/n_i b(C^2 - n_i^2). \quad (61)$$

Accordingly, making use of the easily established identity

$$(2/b)\sum_{i=1}^{\infty}(-1)^{i-1} \cos n_i z/n_i(C^2 - n_i^2) = (1/C^2)(1 - \cos Cz \sec Cb), \quad (62)$$

we have from (42), (49), (61) and (62)

$$T = T_s - (C'/C^2)(1 - \cos Cz \sec Cb) + \sum_{i=1}^{\infty} [A_i J_0(m_i r) + B_i N_0(m_i r)] \cos n_i z, \quad (63)$$

wherein now

$$A_i = D_i \Delta^{-1}[N_0(m_i a) - N_0(m_i a')], \quad (64)$$

$$B_i = D_i \Delta^{-1}[J_0(m_i a) - J_0(m_i a')], \quad (65)$$

$$\Delta = J_0(m_i a) N_0(m_i a') - J_0(m_i a') N_0(m_i a). \quad (66)$$

Correspondingly, from (63) we have

$$T_m \cong T_s - (C'/C^2)(1 - \sec Cb) + \sum_{i=1}^{\infty} [A_i J_0(m_i a_0) + B_i N_0(m_i a_0)], \quad (67)$$

and from (63) and (58)

$$T_a = T_s - (C'/C^2)(1 - \tan Cb/Cb) + [1/b(a - a')]\sum_{i=0}^{\infty} 2(n_i m_i)^{-1} \times \{\sum_{j=0}^{\infty} A_i[J_{2j+1}(m_i a) - J_{2j+1}(m_i a')] + B_i[N_{2j+1}(m_i a) - N_{2j+1}(m_i a')]\} \sin n_i b. \quad (68)$$

Values of Bessel functions for real and imaginary arguments of J_p and for real arguments of N_p are encompassed in Jahnke and Emde's tables. Values of N_p for imaginary argument are not tabulated therein.

However, $N_p = (J_p - H_p)(-1)^{1/2}$ wherein H_p is the Hankel function of order p, values of which for imaginary argument are tabulated; whence N_p is determinable. For purposes of computation, though, it is more convenient to have formulas expressed directly in terms of J_p and H_p. Thus, substituting for N_0 we obtain on combining (64) to (67)

$$T_m \cong T_s - (C'/C^2)(1 - \sec Cb) - \sum_{i=1}^{\infty} D_i\{[H_0(m_ia') - H_0(m_ia)]J_0(m_ir) + [J_0(m_ia) - J_0(m_ia')]H_0(m_ir)\} \times \{J_0(m_ia)H_0(m_ia') - J_0(m_ia')H_0(m_ia)\}^{-1}. \quad (69)$$

If desired, similar equations for T and T_a can be effected without difficulty.

7. A NUMERICAL EXAMPLE.

To illustrate use of the formulas derived in Sections 5 and 6 we consider the following problem: to calculate the temperature in the center of cross section of the rectangular toroidal coil of Run 2 of the experiments of Rogowski and Viewig.[14] Calculation of this same temperature was effected in the writer's [1] earlier paper; whence recalculation affords comparison as to the numerical labor involved and the accuracy attained in calculating coil temperatures on the basis of the actual toroidal geometry (as below) and on the basis of approximating the toroidal coil by a "cylindrical" coil (as earlier).

Given data: Toroidal coil with square cross section, $a' = 4$ cm., $a = 8.4$ cm., $b = 2.2$ cm.; winding comprised of 1416 turns of 0.1 cm. diameter copper wire insulated with lacquer-soaked silk; these turns arranged in coaxial layers, each layer comprised of 40 turns; coil surface temperature 76.1° C., center temperature 122.4° C.; mean temperature calculated from the "hot" or operating resistance 96.7° C.; cold resistance $\approx$ 11 ohms; coil current 3 amperes.

Derived data: Using the given data, we find the volume V of the coil to be 754 cm.^{3} The per cent. conductivity of the copper wire is not stated. Taking it to be 100 per cent., we have (as can be found in any electrical handbook) that $\alpha_0 = 0.00423°$ C.$^{-1}$ and $\alpha_{20} = 0.00393°$ C.$^{-1}$. The temperature at which the cold "resistance" $\approx$ 11 ohms was measured is not stated. We assume 20° C. (68° F.). Then $R_0 = 11[1 - 0.00393 \times 20] = 10.13$ ohms; whence $q_0 = I^2R_0/V = 0.1207$ watts cm^{-3}. Jakob has calculated the thermal conductivities to be $k_r = 0.00506$ watts cm.$^{-1}$ ° C.$^{-1}$ and $k_z = 0.00556$ watts cm.$^{-1}$ ° C.$^{-1}$. In view of the uncertainties that arise in the course of calculating k_r and k_z, these values may be in error by between 5 and 10 per cent. Inasmuch as they differ only about 10 per cent, it may be that use of

[14] W. Rogowski and V. Viewig, *Arch. f. Elek.*, 8, 329–331 (1919–1920).

the average value, $k_r \sim k_z \sim 0.00531$ watts cm.$^{-1}$ ° C.$^{-1}$, whence $k \sim 1$, will serve to minimize the effects of inaccuracies in k_r and k_z (this point is discussed further below).

Computation: Using these derived values of α_0, q_0, and k we find by direct computation:

$$C^2 = 0.0963;\ C' = 30.1;\ C'/C^2 = 312.1;\quad Cb = 0.683;$$

$$n_1 = 0.731;\ n_1 b = 1.571;\ n_1^2 = 0.533;\ \sec(0.683) = 1.288;$$

$$m_1^2 = -0.437;\ m_1 = 0.662i;\ m_1 a' = 2.65i;\ m_1 a_0 = 4.10i;\ m_1 a = 5.56i;$$

$$J_0(2.65i) = 3.419;\ J_0(4.10i) = 12.324;\ J_0(5.56i) = 40.57;$$

$$iH_0(2.65i) = 0.0333;\ iH_0(4.02i) = 0.00695;\ iH_0(5.03i) = 0.00128.$$

These values were calculated on a ten-inch slide rule or taken from Jahnke and Emde's tables. Substituting appropriately in (69) we obtain

$$T_m = 76.1 + 89.8 - (42.3° - 0.0^+) = 123.6°.$$

The terms within the parentheses are those contributed by the infinite series in (69). We note that the series converge so rapidly that generally it is necessary to compute only the first term; the second term is certainly much smaller than the error inherent in the general procedure.

Comparing this calculated value with the corresponding measured value 122.4° C. we find a difference of 1.2° C. or 1 per cent. A calculation similar to that above but using the cited values of k_r and k_z yields 134.6° C., 11.8° C. higher than the measured value. This discrepancy would seem to confirm the remark *re* the effect of averaging the values of k_r and k_z—also to emphasize the necessity of accurate determination of the equivalent thermal conductivities if the formulas are to yield accurate values of temperature.

APPENDIX.

Over the range $-b \leq z \leq b$ a function $f(z)$ that satisfies certain necessary conditions (usually fulfilled by functions encountered in actual practice) can be expanded in a generalized Fourier series

$$f(z) = \sum_{i=1}^{\infty} M_i \cos(n_i z + n_i'),$$

whereof

$$\tan(n_i b + n_i') = C_1/n_i, \qquad \tan(n_i b - n_i') = C_2/n_i,$$

C_1 and C_2 arbitrary constants, and M_i is given by

$$M_i = 2n_i(2n_i b + \sin 2n_i b \cos 2n_i')^{-1} \int_{-b}^{b} f(z) \cos(n_i z + n_i') dz.$$

Illustratively, if $f(z) = C'$ we have

$$M_i = 2C'(2n_ib + \sin 2n_ib \cos 2n_i')^{-1}\int_{-b}^{b} \cos(n_iz + n_i')dz$$

$$= 4C' \sin n_ib \cos n_i'(2n_ib + \sin 2n_ib \cos 2n_i')^{-1},$$

and

$$f(z) = 4C' \sum_{i=1}^{\infty} \sin n_ia \cos n_i'(2n_ib + \sin 2n_ib \cos 2n_i')^{-1} \cos(n_iz + n_i').$$

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(12) T. J. HIGGINS, "Formulas for Calculating Temperature Distribution in Transformer Cores and Other Electrical Apparatus of Rectangular Cross Section," *Trans. Am. Inst. Elec. Eng.*, **64**, 190-194 (1945).

(13) E. H. RAYNER, "Report on Temperature Experiments Carried Out at the National Physics Laboratory," *J. Inst. Elec. Eng.*, **34**, 613-659 (1904-1905).

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HOW TO SAY "NO" IN MATHEMATICS.

BY

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During the last sixty years a new concept has been developing in mathematics which I will call the ability to say "no." Although it may date farther back, for present purposes we may regard it as starting in the 1880's with the work of Heaviside [1] on his operational calculus. This was the subject of much controversy, regrettably.

About 1916 Bromwich [2] and Wagner [3] showed that results similar to Heaviside's could be obtained by more nearly "standard" mathematical methods. Interest in these alternative methods grew and starting around 1925 a number of books [4] on the subject of these and other alternative methods were published. It seems reasonable to assume that if all these different methods give the same results, there must be some unifying principle which brings this about.

In comparing these various methods, it becomes apparent that they are all making use of a new concept in mathematics which is the ability to say "no." This ability enables the discontinuous functions encountered in the solution of many problems to be represented by a single expression, instead of two or more, without any ambiguity. This simplifies and shortens mathematical processes.

Before discussing this concept in more detail, it may be well to reflect for a moment on how similar concepts have often waited for a long time before being admitted into mathematics. First only integers were used and centuries elapsed before fractions were introduced. The negative numbers were quite a problem but they finally found acceptance. The imaginary had similar difficulties and then came the complex numbers. Most people are acquainted with the fact that

[1] Heaviside, Electromagnetic Theory, 3 Volumes.

[2] Bromwich, Normal Coordinates in Dynamical Systems, *Proc. Lond. Math. Soc.* (2), 15, 1916, pp. 401–448.

[3] Wagner, Über eine Formel von Heaviside zur Berechnung von Einschaltvorgange, Archiv. für Electrotechnick, Vol. 4, p. 159, 1916.

[4] The following books illustrate some of these different methods:
Jeffreys, Operational Methods in Mathematical Physics, Camb. Univ. Press, 1927.
Carslaw and Jaeger, Operational Methods in Applied Mathematics, Oxford Press, 1941.
Carson, Electric Circuit Theory and Operational Calculus, McGraw Hill, 1926.
Doetsch, Theorie und Anwendung der LaPlace Transformation, Berlin, 1937.
Gardner and Barnes, Transients in Linear Systems. Wiley, 1942.

Fourier's Series had difficulty in gaining recognition and that Fourier had to arrange for publication of his work when he became Secretary of the French Academy, which was a long time after he first submitted it. Heaviside's work, even at the present time, is still regarded with suspicion.

In order to understand the need for the ability to say "no," we can look at some of the statements commonly used in modern mathematics. For instance, we encounter frequently a statement such as:

Let the function $f(x)$ be defined [5] by

$$\begin{aligned} f(x) &= 0 \quad \text{when} \quad -\infty \leqslant x \leqslant 0, \\ f(x) &= x^n \quad \text{when} \quad 0 \leqslant x \leqslant \infty. \end{aligned} \tag{1}$$

Such a definition is of a type which we might call permissive. In using it we have to know in which part of the range the independent variable lies to decide whether we shall use one or the other of the expressions for $f(x)$. It might seem that this is somewhat trivial. On thinking it over, however, it should be apparent that having two expressions for $f(x)$ and having no way to definitely say that one form should *not* be used in one part of the interval and the other form should *not* be used in the other part of the interval, we have only half stated the definition. We have only said "yes" and have not said "no." To make up for this we rely upon memory to carry us through to a successful result. I think it will be agreed that if the dependence upon memory can be avoided, it will be very desirable. The current trend towards the use of calculating machines also makes the dependence upon memory a thing to be avoided.

One method of approaching this problem of saying "no" is to introduce a symbol which will be carried through all the equations and whose sole purpose will be to indicate at all times that in one part of the interval its value is zero and in the other part of the interval its value is unity. Heaviside used his "unit function" for this purpose and denoted it by the symbol 1. He defined it as a function having the value zero when x is less than zero and having the value unity when x is greater than zero. Other authors have used different symbols. Jeffreys and Carslaw [6] use $H(x)$. I have found it desirable to use the symbol $H(x, x_1)$ to permit the change of the value of the function from zero to unity to take place at any desired point x_1.

[5] The use of the sign $\leqslant$ in connection with ∞ is not standard mathematical procedure. I have used it since it seems to make the definition more definite. For those who prefer to use the sign $<$ in such cases it may be substituted for the sign $\leqslant$ without encountering any difficulty relative to the main discussion in the paper.

[6] Carslaw also uses the form $H(t - x)$ which would correspond to our $H(t, x)$. I think the form which occurs frequently $H(x, 0)$ looks better when thus written than when written $H(x - 0)$ which may be carelessly contracted to $H(x)$.

We thus define $H(x, x_1)$ as follows:

$$\begin{aligned} H(x, x_1) &= 0 \quad \text{when} \quad -\infty \leqslant x \leqslant x_1, \\ H(x, x_1) &= 1 \quad \text{when} \quad x_1 \leqslant x \leqslant \infty, \\ 0 \leqslant H(x, x_1) &\leqslant 1 \quad \text{when} \quad x = x_1. \end{aligned} \tag{2}$$

It is recognized that further discussion of the value of $H(x, x_1)$ at $x = x_1$ may be required on account of the multiple values I have assigned to it.[7] It does not seem necessary to go into this further at the moment since it is possible to go a long ways without having to consider in detail its value at $x = x_1$.

Returning now to equation (1), it is evident that it can be written in the form of a single equation as follows:

$$f(x) = x^n H(x, 0), \tag{3}$$

since the term $H(x, 0)$ automatically makes the right-hand side 0 when x is less than zero and its value is x^n when x is greater than zero. We have thus been able to remove the necessity of $f(x)$ having two different expressions for different parts of the range for the independent variable and to give its value in a single expression. This reduction from two equations to one alone is an attractive feature of the use of $H(x, x_1)$.

As mentioned previously this reduction to a single expression may also be accomplished in many other ways some of which have been used as the basis for operational calculus. To cite only a single example, we may take Bromwich's[8] integral as follows:

$$\phi(p)H(x, 0) = \frac{1}{2\pi i}\int_L \frac{e^{kx}}{k}\phi(k)dk. \tag{4}$$

It will be noticed in the above equation that on the left-hand side we have the factor $H(x, 0)$ which automatically makes the value of the expression on that side zero when x is less than zero. A property of the integral [9] on the right-hand side is that its value is zero when x is less than zero. Thus Bromwich's integral gives us an alternative form for writing in a single equation a function whose value is zero for x less than zero and has specified values for x greater than zero. Hence it gives us again this ability to say "no."

To use it in our example, first let $\phi(p) = p^{-n}$ and obtain

$$p^{-n}H(x, 0) = \frac{1}{2\pi i}\int_L \frac{e^{kx}dk}{k^{1+n}}. \tag{5}$$

[7] A brief discussion of these multiple values for a slightly different function will be found in an article by the author in Journal of The Franklin Institute, Vol. 200, 1925, p. 524, eq. (9).

[8] Jeffreys, Operational Methods in Mathematical Physics, p. 19, eq. 2.

[9] Titchmarsh, Theory of Fourier Integrals, Oxford Univ. Press, p. 6, eq. (1.4.2).

But recollecting that p means d/dx, from this it can be shown that [10]

$$p^{-n}H(x, 0) = \frac{x^n}{n!}H(x, 0), \tag{6}$$

so that

$$x^nH(x, 0) = \frac{n!}{2\pi i}\int_L \frac{e^{kx}dk}{k^{1+n}}. \tag{7}$$

Thus if we write

$$f(x) = \frac{n!}{2\pi i}\int_L \frac{e^{kx}dk}{k^{1+n}}, \tag{8}$$

the function so defined is the same as the function already given by equations (1) and (3).

This may be put in another way by saying that if a function is defined as in equation (1) by two different expressions for given ranges of the independent variable, these can be transformed into a single expression for the function over the whole range of the independent variable by writing it in the form given in equations (3) or (7). In doing this we make use of the ability to say "no."

Whether one uses this ability to say "no" by means of the symbol $H(x, x_1)$ or the Bromwich integral or any other suitable method is somewhat a question of choice and perhaps depends upon the conditions of the problem to be solved. For instance, I have found it possible to use the function $H(x, x_1)$ in the solution of the partial differential equations of mathematical physics involving one, two, three or more dimensions while to date the complex integral has not been applied to more than one dimension. On the other hand, when we consider fractional integration and differentiation, some may feel the complex integral gives results more directly, but this only means that the use of $H(x, x_1)$, in this connection has not been developed to its full extent. Since for the present paper we definitely restrict the discussion to the field of whole integrations and differentiations, it may be fair to state the author's experience that in this field the use of the function $H(x, x_1)$ will do all that the complex integral will do and as of the moment has succeeded in doing more. In a later paper we shall show how in one example, this and other methods of saying "no" used in recent developments of the operational calculus all lead to the same result. We shall therefore not carry the discussion of the complex integral further at the moment but will confine our further discussion to the use of $H(x, x_1)$.

A comparatively simple example has been given of a function which is 0 when x is less than 0 and is given by a definite expression when x is greater than 0. It should be evident that a function can be built up which can have values given by different expressions for different ranges

[10] Jeffreys Operational Methods in Mathematical Physics, p. 19, eq. 4 where the operand $H(x, 0)$ is omitted on each side of the equation.

of the independent variable in a similar manner. As an example suppose [11]

$$\begin{aligned} f(x) &= x^p \quad \text{when} \quad -\infty \leqslant x \leqslant x_1, \\ f(x) &= x^q \quad \text{when} \quad x_1 \leqslant x \leqslant x_2, \\ f(x) &= x^r \quad \text{when} \quad x_2 \leqslant x \leqslant \infty. \end{aligned} \tag{9}$$

Then we can write this in the single form

$$f(x) = [1 - H(x, x_1)]x^p + [H(x, x_1) - H(x, x_2)]x^q + H(x, x_2)x^r. \tag{10}$$

The symbol $H(x, x_1)$ can also be used to make more precise our statement of a function which is defined only over a given interval. For instance, a typical function in the form commonly used might be given as

$$f(x) = x^n, \qquad 0 \leqslant x \leqslant \infty, \tag{11}$$

with no data given as to the value of $f(x)$ for values of x in the range $-\infty \leqslant x \leqslant 0$. If one inquires what the values are in this region, the reply points out that since it is not to be used in this region, the values do not matter. This is hardly a very scientific approach and examples might be cited to show how it has led to confusion and misunderstanding. A much more definite way to proceed would be to say if the values in the region $-\infty \leqslant x \leqslant 0$ do not matter, it is permissible to define them in a suitable manner and the simplest form is to say the function is zero for all such values. Thus we write

$$f(x) = x^n H(x, x_1), \tag{12}$$

and in this form the definition is more precise than that given in (11).

For the case of a finite interval, we would similarly replace the definition

$$f(x) = x^n, \qquad a \leqslant x \leqslant b, \tag{13}$$

by the more definite statement

$$f(x) = x^n[H(x, a) - H(x, b)], \tag{14}$$

which makes the function zero at all points outside the interval in which it is defined.

The function defined by

$$f(x) = x^n \tag{15}$$

would then mean only one thing, i.e. that $f(x) = x^n$ over the interval $-\infty \leqslant x \leqslant \infty$ and not all the different functions which it represents in mathematics now by restricting its definition to certain intervals.

[11] Objection might be raised to the multiple values of $f(x)$ at the ends of the intervals. This is not important for the present discussion and may be avoided if desired by changing some of the $\leqslant$ signs to $<$.

This concept of how to say "no" may seem so simple that on the one hand one wonders why it has not been introduced long ago and on the other hand one may wonder what use can such an elementary notion possibly have. The answer to the latter question may perhaps be made by a quotation from Jeffreys [12] discussing the operational methods of Heaviside as follows—"but it is certain that in a very large class of cases the operational method will give the answer in a page where the ordinary method takes five pages and also that it gives the correct answer when the ordinary methods, through human fallibility, are liable to give a wrong one." The above quotation would apply equally well if we substituted for the words "the operational method" the words "the ability to say 'no'" since as has been stated previously, the Heaviside method makes use of this ability.

If we accept this ability to say "no" and proceed to use it in applications, we do not go very far before we encounter another concept that has been struggling for recognition in mathematics for even a longer time. This is the impulse function. We encounter it immediately in problems involving differentiation when we make use of the symbol $H(x, x_1)$. For instance if

$$f(x) = x^n H(x, x_1), \tag{16}$$

then we might write in the usual manner for the differential coefficient of a product.

$$df(x)/dx = nx^{n-1}H(x, x_1) + x^n dH(x, x_1)/dx. \tag{17}$$

Here we have to interpret $dH(x, x_1)/dx$. It is obvious from (2) that its value is zero everywhere except at $x = x_1$, and there is agreement among all writers on this. But we do not find such agreement on its value at $x = x_1$. Jeffreys [13] gives its value as zero. Carslaw [14] on the other hand recognizes its value at this point as being infinite in such a way that its integral from $x_1 - \epsilon$ to $x_1 + \epsilon$, $(\epsilon \to 0)$, is equal to unity. It is interesting to note that Carslaw relegates this discussion of the impulse function to an appendix and further states that methods using it lack rigor.

The following quotation from Giorgi [15] is of interest in this connection.

"At first sight it may appear that the use of these impulsive functions is strange and illegitimate, as it involves the consideration of actual infinitesimals and infinities. But, in fact, it is very useful because it simplifies the formulae very greatly; in fact not one of the most rigorous

[12] Jeffreys, Operational Methods in Mathematical Physics—introduction.

[13] Jeffreys, p. 19, eq. (5).

[14] Carslaw and Jaeger, p. 251, eq. (2).

[15] Giorgi, On the Functional Dependence of Physical Variables, *Proc. Int. Congress of Mathematics*, Toronto, Vol. 2, 1924, pp. 31–56.

writers on dynamics has refrained from introducing the impulsions (impulsive functions). As regards the theoretical standpoint, it is to be remarked that actual infinitesimals and infinities may be introduced with perfect rigor as a class of non-Archimedean numbers involving special postulates or what amounts to the same thing, we may say that all formulae containing improper functions are formula wherein the sign of Lim is understood so that $Fu(t)$ (the impulse function) and similar symbols may be regarded as a kind of shorthand notation."

There are numerous forms in which the Impulse Function is used in the literature and we may give a few as examples.

Heaviside uses it in several forms and we may list three of them showing their relation to our notation $dH(x, 0)/dx$ for the impulse function at $x = 0$.

$$(1)\quad dH(x, 0)/dx = p1, \tag{18}$$

$$(2)\quad dH(x, 0)/dx = x^{-1}H(x, 0)/\lfloor -1, \tag{19}$$

$$(3)\quad dH(x, 0)/dx = \frac{1}{\pi}\int_0^\infty \cos mx dm. \tag{20}$$

Campbell and Foster [16] give it in the form

$$dH(x, x_1)/dx = S_0(x - x_1) = \lim_{a\to 0} a^{-1}\exp\left[-\pi(x - x_1)^2/a^2\right]. \tag{21}$$

Dirac [17] gives it in the form

$$dH(x, 0)/dx = \delta(x). \tag{22}$$

Carslaw [18] gives it in the form

$$\begin{aligned} dH(x, 0)/dx = \delta(x) &= 0, && x \leqslant 0 \\ &= 1/\epsilon, && 0 < x < \epsilon \\ &= 0, && x \geqslant \epsilon. \end{aligned} \tag{23}$$

Although Jeffreys specifically states $p1 = 0$, we may, nevertheless, speculate on the possibility of using Bromwich's integral in the form

$$dH(x, 0)/dx = \operatorname*{Lim}_{n\to 1} p^n H(x, 0) = \operatorname*{Lim}_{n\to 1} \frac{1}{2\pi i}\int_L \frac{e^{kx}dk}{k^{1-n}}. \tag{24}$$

The number of possible representations of the impulse function is unlimited. Heaviside has an interesting discussion [19] pointing out that from the solution of any electromagnetic problem involving the field

[16] Fourier Integrals for Practical Applications, Bell Telephone System Tech. Publication, 1931.

[17] Dirac, Quantum Mechanics, 1st Edition, p. 64.

[18] Operational Methods in Applied Mathematics, Carslaw and Jaeger, Oxford Press, 1941, p. 251.

[19] Electromagnetic Theory, Vol. 2, pp. 93 and 99.

due to a point charge, we may derive an expression for the charge density and since this is zero everywhere except at the charge where its integral must be Q the value of the charge, this gives us, therefore, an expression for the impulse function.

These illustrations should be enough to show that the impulse function has a definite place in the literature. Of course, the mathematician will say that some of the expressions listed above "do not exist." However, we should recognize that the term "do not exist" when used thus is simply a technical term to say that the rules that mathematicians set up to govern their operations fail when they are applied to such expressions. It would appear much better if instead of using the term "they do not exist" the mathematicians used the phrase "we do not understand them yet" as this might encourage some of the younger men to study them and broaden mathematics so as to develop guiding rules for the use of such expressions so that they may be safely used by all. The present statement that they "do not exist" seems to give rise to an attitude that it would be useless to attempt a study of such expressions since they have no existence anyway and this tends to discourage younger workers from really making a contribution in this field.

This outlines briefly the situation with regard to the impulse function, and perhaps explains why it is not more widely used. This is unfortunate since a knowledge of the impulse function in all its various forms would allow us to solve problems on sight which are quite laborious using the standard mathematical methods.

Our purpose in this paper is to emphasize the two concepts which we have discussed as ones which have a large field of use in mathematics. One of them, the ability to say "no," is new and has not been named as such in the literature although it has been used in different forms by various authors. The other, the impulse function, has been widely used but its use is considered to be lacking in rigor. Since we have desired to stress the importance of these ideas as concepts, it seemed desirable to omit from this paper any detailed mathematical examples illustrating the use of these concepts. These have been relegated to later papers. One of them will show how by use of the ability to say "no" we can make integration and differentiation reversible operations. We can then extend the same idea to differential equations as can be shown by examples and a reference will be given to work showing how it can also be used in the case of partial differential equations to obtain solutions which up to now have not been obtained by the standard mathematical methods. The other paper will show how some of the more recent developments of operational methods using integrals such as those of Bromwich, Carson, Carslaw, Gardner, and Barnes all make use of this same ability to say "no" although these authors do not seem to be conscious of the fact that they are using it.

Based upon these later papers and the discussion in the body of this paper, certain advantages are obtained from these concepts which it may be well to summarize here.

1. The use of the ability to say "no" allows us to define in a single expression a function which is defined by different forms in various parts of the interval in which it is defined. See equation (10). This is in contrast with the standard mathematical method in (9) of giving two or more forms which must be used depending upon what part of the part of the interval the value of the function is required.

2. The use of the ability to say "no" allows us to make more precise our definition of a function which is only defined over a certain interval. Here the approach is to use the ability to say "no" to ensure that the value of the function we use is zero at all points outside the interval in which it is defined. See equation (14). This can be contrasted with the standard mathematical method in (13) of leaving it undefined outside of that interval.

3. The use of the ability to say "no" permits us to make integration and differentiation reversible operations. This, of course, is an enormous advantage in eliminating integration constants which appear in the "classical" method and cause considerable difficulty in their determination. This will be shown in a later paper.

4. The use of the ability to say "no" to eliminate constants of integration may be extended to differential equations. These can be written in such a form that they contain sufficient information in themselves so that no additional information is required as to initial conditions, etc., to obtain the solution for a given problem.[20] In comparison with this, the "classical" method requires n initial conditions for the solution of a nth order differential equation.

5. The use of the ability to say "no" may be extended to the partial differential equations of mathematical physics and their solution found for the conditions corresponding to the Green's Function with given conditions on the boundaries without any difficulty in evaluating the coefficients in the Fourier Series or Integrals in which the results are expressed.[21] In contrast with this, the "classical" method requires the evaluation of an infinite number of unknown constants.

6. The use of the ability to say "no" may be extended to the problem of fractional integration and differentiation and is again of enormous advantage in eliminating integration constants. These integration constants present quite a problem in the "classical" approach to fractional differentiation and integration which has not yet been satisfactorily resolved. This use of the ability to say "no" has not

[20] This will be shown in more detail in a later paper but an example of its use will be found in a paper by the author in JOURNAL OF THE FRANKLIN INSTITUTE, Oct., 1944, Vol. 238, p. 248.

[21] Ibid. Here $H(x, x_1)$ is used for various coordinates throughout the paper. Its introduction for one coordinate ξ can be seen in equations (4.4), (4.5), and (4.6).

been discussed in this present paper but will be covered in a later one. It is mentioned here, however, since it so closely parallels the same situation with regard to whole differentiation and integration pointed out in Item 3.

7. In all these uses of the ability to say "no" by means of the symbol $H(x, x_1)$, its first order derivative $dH(x, x_1)/dx$ and at times higher order derivatives are also involved. These derivatives are impulse functions and their use has been questioned by many mathematicians some of whom have gone so far as to say they "do not exist." They have been used successfully, however, by many others and in several different forms, a few of which we have listed in the body of the paper. In view of the great advantages obtained from their use, it would seem highly desirable that considerable study be given to these impulse functions or derivatives of $H(x, x_1)$ in order that agreement may be reached on rules guiding their use. Such rules would permit the wider use of the impulse function by many people with resulting benefits.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

COLOR STANDARDS FOR AVIATION LIGHTS.

Airplane operations at night are controlled by radio and lights. So far as it is possible to do the work with lights, the pilot prefers this method because it is more direct. The controlling of flight involves a variety of problems such as prevention of collision in the air, navigation of aircraft along airways, identification of landing areas, guidance of aircraft along approachway and runway during landing and take-off operations, and direction of aircraft on the ground while taxying.

Lights for so many purposes must be of different colors. The present colors in use by the Army, Navy, and Civil Aeronautics Administration, require the use of two types each of red, yellow, and green, and one type of blue and lunar white, as well as units without colored ware. In order to prevent lights of different colors from being confused, a careful restriction of the variation of the several colors is necessary.

The Bureau has completed the standardization of 14 sets of filters to be used in controlling the colors of aviation lights. These filters were prepared by the Corning Glass Works with the technical assistance of the Bureau. Ten sets of 18 glasses each have been standardized for the Army Air Forces. The remaining sets, which contain 9 filters each, will be used by the Civil Aeronautics Administration. Many of these filters are already in use by inspectors of glass and plastic ware that are being purchased by the Government for use in lights to be mounted on aircraft or set up at landing fields, including some located close to the battle fronts.

* Communicated by the Director.

SUBSTITUTE MOTOR FUELS.

As part of an investigation of substitute motor fuels, conducted at the Bureau for the Foreign Economic Administration of the Office for Emergency Management, a study has been made by Donald B. Brooks of the comparative power, thermal efficiency, and combustion performance of a series of non-hydrocarbon fuels and of a reference gasoline. The non-hydrocarbons, used in forming either neat or blended fuels, included ethyl alcohol, diethyl ether, acetone, and butanol. Tests were made in a precision single-cylinder variable-compression engine, direct

connected to an a-c. dynamometer, which maintained essentially constant speed.

Tests made at constant compression ratio, simulating the use of these substitute fuels in present engines, designed for operation on gasoline, and giving no more than trace knock on any of the fuels, showed that the power developed by the non-hydrocarbon fuels was from 2 to 5 per cent. greater than that developed by gasoline. Their thermal efficiencies were slightly higher than that of gasoline at lean mixture ratios. Little difference was noted in the combustion performances of the fuels.

When each fuel was tested at the compression ratio at which it gave trace knock at best-power mixture ratio and optimum spark advance, simulating its use in an engine designed to take full advantage of its anti-knock qualities, materially more power and higher thermal efficiencies were developed by the non-hydrocarbon fuels than by gasoline. The power increases ranged up to one-fourth, the highest thermal efficiency being 36 per cent., as compared with 28 per cent. for gasoline. Under these conditions also, the combustion performances of the fuels were closely related.

The results of these tests are given in detail in the *Journal of Research* for July (RP1660). They may be summarized by saying that in the absence of knock, the performance of the engine is determined by the specific fuel heat-input, without respect to the nature of the fuel furnishing the heat. The knock rating of the fuel governs the compression ratio that may be employed, and thereby determines the power and thermal efficiency that may be obtained.

THERMAL LENGTH CHANGES OF ZIRCONIA.

Zirconia (ZrO_2) is obtained from zircon ($ZrSiO_4$), a mineral which was well known in the early Christian era as the gem stone Hyacinth. Zirconia has been used for some years as an opacifier in enamels, and to some extent in glazes. Its application as a refractory has not been advanced to an appreciable extent even though it is available in high purity (96 to 99 per cent. ZrO_2) and at a low cost when compared, for example, with thoria or beryllia. It has a very high resistance to heat (the melting point is believed to be about 2,700° C. or possibly higher), it is chemically stable at ordinary and at elevated temperatures, and it is hard (6 on the Mohs scale). Shapes made of zirconia are, however, inclined to disintegrate rather rapidly in service. This disintegration—manifested by progressive cracking until the entire shape has broken up into small, roughly cubical pieces—is caused by large and very rapid changes in volume that accompany the reversible inversions of the low temperature monoclinic form and the tetragonal form of zirconia.

Thermal expansion and differential heating tests showed that the change from monoclinic to tetragonal form, during heating, occurs between 1,100° and 1,400° (depending somewhat on the purity of the zirconia), causing the material to shrink and to absorb heat (endothermic reaction). During cooling, the tetragonal reverts to the monoclinic and causes a rapid expansion, beginning sharply at some temperature between 980° and 1,040° (depending on the purity) and accompanied by an evolution of heat (exothermic reaction).

Published information indicated that the addition of certain oxides to zirconia would alter its crystal form to a "stabilized" cubic lattice, which is not subject to inversions, and thereby would eliminate the destructive volume changes during heating and cooling. In an investigation by R. F. Geller and P. J. Yavorsky reported in the July *Journal of Research* (RP1662), the oxides of cerium, yttrium, silicon, magnesium, and calcium were added to zirconia of 99 per cent. purity, the specimens made of these mixtures were heated at various temperatures ranging from 1,450° C. to 1,950° C. and the thermal length changes were determined during both heating and cooling between room temperature and a maximum of 1,700° C.

The results show that the irregular thermal length changes accompanying phase transformations in zirconia may be prevented by changing the crystal to the stable cubic form. This was accomplished by (a) 11.5 and 15 per cent. additions of Y_2O_3 and heating at 1,700° or higher; (b) 8 and 15 per cent. additions of MgO and heating at 1,550° or higher, though in this case, irregularities were suppressed only for the range from room temperature to 1,200°; and (c) 5, 6, 8, and 15 per cent. additions of CaO and heating at 1,550° or higher, but additions of more than 6 per cent. caused the specimens to be very porous and proportionately weak. In all cases, however, the expansion during heating and contraction during cooling were relatively high. Also, zirconia has a low thermal conductivity relative to materials of high thermal expansion, such as alumina, magnesia, and beryllia (unpublished data). Consequently, a high resistance to thermal shock cannot be expected of the stabilized product even though it is not subject to the structural disintegration characteristics of the commercially pure material.

NEW THERMAL INSULATING MATERIALS.

A surprisingly large number of new thermal insulators are arriving at the Bureau for test. These include some new expanded plastics in foam form which weigh not much more than one pound per cubic foot. Other materials can be formed in place in irregular spaces. It appears likely that marked practical advances in this development will appear after the war.

ACETYL DERIVATIVES OF CERTAIN HEPTOSES, OF GULOSE, AND OF LACTULOSE.

The acetyl derivatives of the sugars are among the most important sugar compounds from the standpoint of general usefulness for analytical purposes, in synthesis, and in the elucidation of structure. They are uniquely adapted to the study of the Walden inversion because the configuration of each asymmetric carbon is known, and the positions of the various atoms in space are more or less fixed by the presence of a ring structure. The intimate knowledge of the relative positions of the atoms and groups within the molecule permits experimental confirmation of some of the theories concerning the stereomeric factors necessary for replacement reactions.

In previous publications Harriet L. Frush and Horace S. Isbell of the Bureau's polarimetry section have shown that cyclic orthoacetates are formed from acetylglycosyl halides when one of the acetyl groups can approach the face of the glycosyl carbon opposite the halogen. The preparation of a wide variety of acetyl derivatives of the sugars was undertaken in order to obtain additional information concerning the formation of ortho esters and the role of acetyl groups in the replacement reactions of the acetylglycosyl halides. In RP1663 in the *Journal of Research* for July, the mechanism for the replacement reactions is reviewed briefly by the same authors, and experimental details are given for the preparation of several new acetates of the heptoses, and for acetates of D-gulose, D-glucoheptulose, and lactulose. The optical rotations of the new compounds are reported and compared with the optical rotations of related substances.

EFFECT OF PRESSURE ON THE MELTING OF CRYSTALLINE RUBBER.

The effect of hydrostatic pressure on the melting of crystalline rubber has been the subject of a brief investigation by Lawrence A. Wood, Norman Bekkedahl, and Ralph E. Gibson, extending to pressures above 1,000 atmospheres. With a particular sample of stark rubber, it was found possible to raise the temperature of melting, as determined by the disappearance of birefringence, from about 36° to 70° C. by the application of a pressure of 1,170 bars ($1{,}170 \times 10^6$ dynes/cm^2). The results, including observations at intermediate pressures, can be represented adequately by the equation,

$$\log_{10}(p + 1{,}300) = 5.9428 - \frac{875}{T}.$$

BRAZIL'S RESEARCH FOR INCREASED RUBBER PRODUCTION.

It is Brazil's desire to regain her former prominent position as one of the important rubber-producing countries of the world. However,

the methods that are used to extract the latex from the wild rubber trees of the jungles of the Amazon Valley and the processes for coagulating the rubber are much too primitive, slow, and costly to compete with the newer methods and more highly-developed and cultivated rubber trees of the Far East. Brazil has recently realized that the cultivation of rubber trees in the Amazon Valley is necessary toward the solution of the problem, but that before this could be done much research would have to be performed because agricultural conditions in the Amazon Valley differ somewhat from those in the Far East.

A leaf-disease very harmful to rubber trees is prevalent in South America and must be conquered before plantations can be successful. Brazil has many types of trees which give different types of rubber having different characteristics, and it was apparent that a physico-chemical research and testing laboratory would be necessary to evaluate all the various types of rubber. The Brazilian Government, therefore, established, in 1941, a research institution called the Instituto Agronomico do Norte near the city of Belem do Para, which is located in the rubber-growing area near the mouth of the Amazon River. The purpose of this institute is to study the problems of botany, pathology, physiology, entomology, technology, and economics connected with all agricultural processes and products of the Amazon Valley. This, of course, includes problems connected with the production of rubber.

This institute, the I. A. N., has been working in close cooperation with the U. S. Department of Agriculture and the Bureau in the United States, and also with the Ford Rubber Plantation in Brazil. A tree has been developed that will give a much greater yield of rubber than the average wild rubber-tree of the jungle. This tree can also be made resistant to the leaf-disease. It is produced by grafting a bud from a tree that is known to give a good yield of latex, to one having a strong root-system. This bud is added near the ground and forms the new trunk, the part of the tree that is tapped for latex. Then in order to give the tree good resistance to leaf-disease a third strain of tree which is not susceptible to the disease is added by topbudding at a height of 6 or 7 feet to form a new crown. With the development of this improved tree, the I. A. N. expects that the Brazilian Government will finance and begin the planting and cultivating of rubber trees in various sections of the Amazon Valley, and that later small acreages will be turned over to individual natives or families after they have been given sufficient instruction in the improved methods of rubber-growing and processing. With a good system of rubber research and technical schools established in the Amazon Valley, Brazil should be able to increase her rubber production and compete favorably in price and quality with both the natural rubber of the Far East and the synthetic of the United States.

Norman Bekkedahl of the Bureau, who served as the first chief of the Instituto's rubber laboratory (*Technical News Bulletin 337*, May

1945) has prepared a technical discussion of this subject for publication in "India Rubber World," and a more popular article that will appear in the "Scientific Monthly."

COMMERCIAL STANDARD FOR PREFABRICATED HOUSES.

Commercial Standard CS125-45, Prefabricated Homes, which has just been released, is designed to improve the quality of construction for prefabricated dwellings. Although construction methods and materials will vary, this performance standard should serve as a dependable basis for the guidance of purchasers, building inspectors, manufacturers, and mortgagees.

This voluntary standard was proposed in November 1943, by the Prefabricated Home Manufacturers' Institute. A preliminary draft was submitted to a large number of organizations for their review and comment. Following adjustments made in accordance with the consensus of suggestions received, the proposed standard was circulated to all known interested manufacturers, technical organizations, lending agencies, testing laboratories, and building officials, for comment. The standard was then adjusted in accordance with the composite comment and circulated to the trade for written acceptance.

The standard provides minimum requirements for 1-, $1\frac{1}{2}$-, and 2-story prefabricated houses. It covers structural strength of the various component parts, light and ventilation, and recommended requirements for foundations, chimneys, heating, plumbing, insulation, and electrical wiring. It includes general requirements for material, workmanship as provided by the manufacturer, erection at site, assembly of prefabricated units, and protection during transportation and erection.

Printed copies of CS125-45 may be purchased from the Superintendent of Documents, Government Printing Office, Washington 25, D. C., at 5 cents each.

THE FRANKLIN INSTITUTE.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The library and reading room are open on Mondays, Tuesdays, Wednesdays, Fridays from nine o'clock A.M. until five o'clock P.M., Thursdays from two until ten o'clock P.M., Saturdays from nine o'clock until twelve noon.

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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Dermatome for the Study of Wound Healing.—M. H. Ross and C. H. Ludwig. In the study of experimental wound healing the greater the reproducibility of the injuries the more valuable the data. In order to obtain wounds of constant area and depth in rabbits' ears, a special dermatome was designed.

This dermatome (see Fig. 1) consists of a hollow brass tube that has one end beveled to a sharp edge so that it becomes a circular knife.

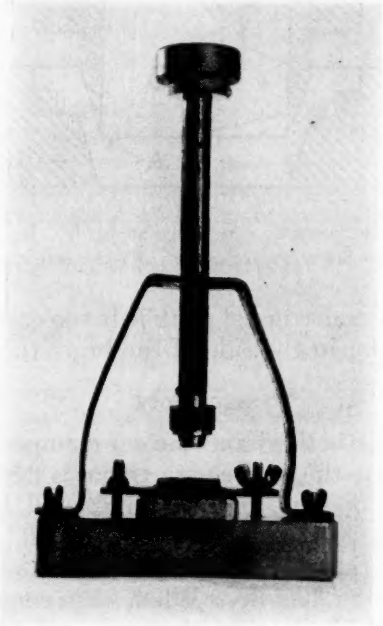

Fig. 1. Dermatome for the study of wound healing.

Within the tube is installed a tool-steel blade at an angle to the horizontal plane; the edge of this blade lies flush with the base of the circular blade. The radial knife within the circular one undercuts the tissue incised. One complete turn of the tube cuts both the outer and the under parts of the tissue. To obtain uniform depth, regardless of pressure, a collar is fitted above the knife edge and this in turn engages the grooved lip of an orifice in the metal clamp through which the animal's ear is incised (see Fig. 2). Changes in depth of incision can be accomplished by raising or lowering the collar on the knife. The

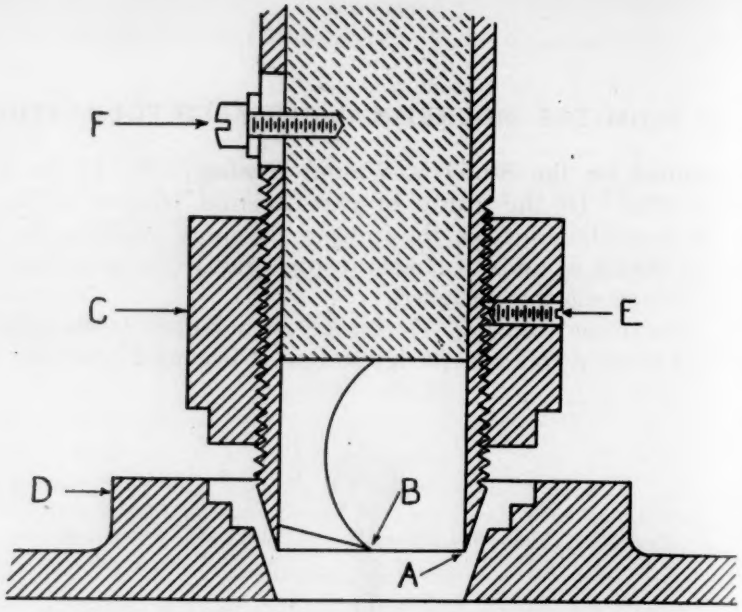

FIG. 2. Cross-section of dermatome. A—Circular blade. B—Radial blade. C—Collar. D—Matrix. E—Set screw for collar. F—Set screw for radial blade.

undercutting knife is constructed so that it too can be raised or lowered by a set screw and slot in the side of the brass tube.

TECHNIQUE.

The animal is anesthetized and the ear clamped down by the matrix in such a manner that the underlying tissue is flat. Only that portion visible through the opening of the clamp will be cut by the knife. Areas that contain large vessels are to be avoided. The tube is lowered so that it fits smoothly into the grooves of the matrix; pressure is applied in turning the handle. When one complete turn has been executed, the knife is lifted and with it comes the flap of tissue. Slight bleeding can be controlled by the application of coagulating materials.

The entire dermatome, which is mounted on a wooden platform and supported by two metal posts, is easily disassembled for sterilization.

BOOK REVIEWS.

THE CHEMISTRY OF ACETYLENE, by Julius A. Nieuwland and Richard R. Vogt. 219 pages 15 × 24 cms. New York, Reinhold Publishing Corporation, 1945. Price $4.00.

Although used for many years as a means for illumination, a tool in industry and as a constituent or product in chemistry, acetylene today stands on the threshhold of even greater usefulness. It is time, therefore, that an accounting be made, reviewing knowledge up to the present, so that a working basis can be had for progress in the future, and at the same time to provide a means for enlightening those whose activities may be along a wholly different line and yet may have border-line significance. This American Chemical Society Monograph is intended to fill this need. It is a brief but fairly complete account of the preparation, properties and reactions of acetylene along with an extensive classified bibliography. Its arrangement is based entirely on a classification of the methods of preparation and type reactions of acetylene, rather than on considerations of historical sequence or industrial status. Bibliographies follow each topic.

The work is divided into five parts. The first deals with physical properties, preparation and purification; the second is on metallo-derivaties; the third is on the substitution of non-metal atoms or radicals for the hydrogen of acetylene; the fourth is on the addition of non-metallic elements and compounds to acetylene; and the fifth is on the polymerization and condensation with carbon compounds. Most literature coverage ends with the year 1938.

The presentation is quite complete. It is an exhaustive inquiry into the literature coupled with a classification and method that should be most useful.

R. H. OPPERMANN.

THE CHEMICAL PROCESS INDUSTRIES, by R. Norris Shreve. 957 pages, tables, drawings and illustrations, 15 × 23 cms. New York, McGraw-Hill Book Co., Inc. 1945. Price $7.50.

Part of the working knowledge of a great many engineers is required to be a general conception of basic manufacture. This is true in cases where the application of the particular science is in a variety of industries over a period of time or where it is in one industry all of the time. In the latter situation there are time and opportunity to become acquainted with unit processes in the manufacture, but where the engineer is called to give service to first one industry, then another, he is handicapped if at least a broad knowledge is not had.

The book at hand is meant primarily for the student in chemical engineering, to be used as a text while studying other subjects. It may also be used as a reference for chemical engineers, and for engineers generally. The objectives sought are to present a cross section of the manufacturing procedures employed by modern chemical industries largely separated into their unit processes and unit operations through the help of flow charts. These industries not only involve the manufacture of chemicals as such, but they embrace many manufacturing processes based on important chemical changes and these naturally include a considerable variety of operations based on data and principles from other branches of science and engineering.

There are 34 chapters each devoted to a process or industry. The order of chapters is largely from a teaching viewpoint—frequently used while the student is taking physical chemistry and organic chemistry simultaneously,—so that the sequence is from the simple to the more complex. Hence, such a chapter as sulphuric acid involving applications of equilibriums and certain other phases of physical chemistry, is placed after the more purely descriptive chapters pertaining to water, fuels, distillation of coal, gases and carbon. After this there are chapters devoted to explosives and chemical warfare; paint, varnish, lacquer; leather, gelatine and glue; oils, fats and waxes; fermentation industries, the petroleum industry; intermediates, dyes and their applications.

The presentation of the processes around the flow sheets and the energy changes leads to a logical following through of a connected series of unit operations and unit processes. The flow charts are valuable too in directing thought along cost lines of the product, although no attempt is made in cost determination. The book gives quantitative engineering aspects and leads to thinking from raw materials to salable products. It is useful as a text for students, and a reference for chemical and other kinds of engineers.

R. H. OPPERMANN.

MAKING PATENT DRAWINGS, by Harry Radzinsky. 96 pages, illustrations, 19 × 26 cms. New York, The Macmillan Company, 1945. Price $3.00.

A patent consists, in addition to the certification or grant, of the drawings, the specifications, and the claims. The patent drawings must be the true picture of the device, article, or element sought to be patented. It must show the parts of the device in their operative positions and, if possible, it must show how they perform the function and achieve the result sought by the invention. All drawings must conform to the exact style prescribed by the United States Patent Office.

This book provides a complete handbook on the preparation of patent drawings. Its purpose is to teach the special technique of patent drawing to those who are familiar with conventional drafting practices. It gives explicit instructions for shading in the form peculiar to patent drawings, for the proper placement of numerals and figures, and all other special technique in making the required drawings both for patents and for trademarks. It shows, step by step, how the patent drawings for both simple and complex inventions are made, with many illustrations reproduced from actual drawings from the Patent Office. It gives much practical advice on the use of drawing materials and on drafting techniques, and shows how to avoid faults or defects in drawings which might cause them to be rejected by the Patent Office.

The author has been a practicing patent attorney for more than twenty-five years, and has himself made several thousand patent drawings. The book is thoroughly illustrated not only with his own drawings but also with illustrations made especially for this book by a well known patent draftsman.

D. COLLINS.

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Transmission Lines, Antennas and Wave Guides, by Ronold W. P. King, Harry R. Mimno, and Alexander H. Wing. 347 pages, drawings, 14 × 23 cms. New York, and London, McGraw-Hill Book Company, Inc., 1945. Price $3.50.

Calcium Metallurgy and Technology, by C. L. Mantell and Charles Hardy. 148 pages, illustrations, 15 × 24 cms. New York, Reinhold Publishing Corporation, 1945. Price $3.50.

Television Programming and Production, by Richard Hubbell. 207 pages, illustrations, 16 × 24 cms. New York and Toronto, Murray Hill Books, Inc., 1945. Price $3.00.

CURRENT TOPICS.

ARMY AND NAVY NOTES.

Petroleum Investigations in Alaska in 1944.—The Geological Survey, Department of the Interior, conducted intensive investigations of some of Alaska's oil possibilities in 1944, according to a statement by the Director of the Survey, William E. Wrather, to Secretary of the Interior, Harold L. Ickes.

Indications of oil in Alaska are widespread and have been known for many years, although investigations of potential oil areas by the Geological Survey have been deferred in recent years because of even more pressing duties. Only a small part of the rocks that may contain oil has been tested; the possibility of the presence of substantial reserves has not been adequately determined, although certain areas are worthy of further investigation. One small oil field was developed through drilling by private organizations, but it has been inactive since 1933.

War demands have caused a heavy drain on petroleum supplies in the Pacific area, thus increasing the desirability of finding oil in Alaska. Large quantities of petroleum products must be shipped to Alaska, although the Canol project can supply part of Alaska's growing needs. Development of petroleum production in the Territory would free both imported petroleum products and shipping for use elsewhere.

R. H. O.

Automotive War Production Reached New High Level in 1944.—In 1944, while American forces blasted their way into France, Germany and the Philippines, the automotive industry established a new all-time high war production record by turning out $9,320,000,000 worth of armament during the year, or more than $1,000,000 worth every hour.

This was announced by George Romney, managing director, Automotive Council for War Production, as the Council marked its third anniversary since its formation after Pearl Harbor.

The total value of automotive war production this year was approximately eight per cent. over that of last year, and brought the grand total produced in former automotive plants since Pearl Harbor to $23,000,000,000, with orders on hand amounting to $11,000,000,000 to be filled in 1945.

Automotive-produced engines for land vehicles, aircraft and marine use were turned out literally by the carload. A grand total of 3,100,000 power plants of all types was produced between the beginning of the war program and the latter part of 1944. Four companies turned out 191,000 engines for Liberators, Flying Fortresses and military cargo planes. Another built over 50,000 engines for PT boats, motor torpedo gun boats and fighter planes. Two others shipped a wartime total of 104,000 engines for invasion craft. Still another delivered more than 15,000 medium tank engines.

Ranging from quarter-ton jeeps to mammoth 10-ton cargo trucks, the industry has produced 2,240,000 military motor vehicles since the beginning

of the military production program. One company recently delivered its 150,000th heavy-duty truck.

The industry has produced 4,700,000 guns, ranging from calibre .30 carbines to giant "stratosphere" anti-aircraft cannon. One company recently shipped its 1,000,000th infantry machine gun.

"This vast output results from the application of individual skills and resources developed during peacetime to the gigantic task of production in competition with our country's enemies," Mr. Romney pointed out. "In the war program, more than 500 automotive manufacturers in 31 states, who were competitors before Pearl Harbor, worked as a production team, exchanging technical data and cooperating to overcome production bottlenecks. In addition, prime contractors were assisted by sub-contractors in 1,375 cities in 44 states. Sixty-three per cent. of the latter are comparatively small businesses, employing less than 500 persons, but their contribution is great. Some small plants furnish as much as 98 per cent. of a finished product. The completed output represents a triumph of integration between prime and sub-contractors."

Mr. Romney also pointed out that contract prices of war equipment produced by the automotive industry, exclusive of transport vehicles, have decreased approximately one-third since Jan. 1, 1942, according to computations from price indexes compiled by the War and Navy Departments. This decline, he said, does not take into account additional savings to the government resulting from lump sum refunds or from renegotiations by price adjustment boards. Among the greatest price reductions is the item of guns of all types, down 56 per cent., with small arms, including machine guns, down 64 per cent.

"These price reductions are due mainly to the application of the industry's mass production methods and advanced manufacturing techniques to the military products," Mr. Romney concluded.

R. H. O.

Anti-Blackout Flying Suits. (*Compressed Air Magazine*, Vol. 50, No. 2.)—If you swing a bucket of water in a circle, centrifugal force keeps the liquid from spilling. In a similar manner, when the pilot of a fighter plane makes a pull-up or abrupt turn at high speed, centrifugal force causes the blood to flow to the lower parts of his body and holds it there. This pull against gravity is known in air-force medical circles as the "G" effect. At "5G," or five times the normal pull of gravity, many aviators "black out"—that is, they lose their vision because the heart is unable to pump oxygen-carrying blood to the brain. At "7G," a 150-pound pilot weighs more than half a ton, and his 4-ton fighter plane is heavier than some bombers. His blood grows as heavy as iron and "pools" in his feet and legs. Sometimes he loses consciousness, as well as power to see.

To offset this phenomenon, pilots of our P-51 and P-47 fighter planes now wear pneumatic pants, known as a G-suit. This equipment applies counteracting pressure to the legs and lower abdomen during rapid turns, and sufficiently offsets the G-effect to prevent the dimming of vision. A flier thus protected has an obvious advantage over enemy pilots who are not similarly equipped, and he can out-maneuver them readily. The G-suit has been used with success by aviators of the Eighth and Ninth air forces in Europe, and news of it was withheld until it was known that the Germans were aware of its details.

The G-suit resembles tight, high-waisted pants, with built-in suspenders and girdle. It has air bladders over the abdomen, thighs, and calves, and these are distended by means of the aircraft's pump that supplies compressed air for instrument operation. The pilot puts the suit on just before going aloft and can wear it partly open until he has need of it. Then he closes it by zippers, plugs the air line into a connection, and an automatic valve does the rest. The latter inflates the bladders when the force of gravity rises above 2G and deflates them when it drops below that figure, either function requiring about two seconds. Although the G-suit theoretically gives the flier an extra tolerance of only 1.9G, fighter pilots report that it actually makes blacking out impossible. They also find that there is a great reduction in fatigue which commonly follows combat maneuvers. The earliest practical suits were developed by the Canadian and Australian air forces. The pioneer work in Canada was done by Dr. William R. Francks, formerly of the University of Toronto, and the suit that he invented in 1939 in conjunction with the Banting Institute of that school and the RCAF was designed to use either water or air as a fluid to build up external pressure to balance the internal forces set up by increased gravity. Dr. Francks and some of his associates at the Banting Institute served as guinea pigs for the initial tryouts in a giant "whirligig" that simulated conditions that would be met in an aerial dog fight. The first test in actual warfare came in 1942 when carrier based fighter planes of the British Fleet Air Arm speedily swept aside enemy opposition at Oran in French North Africa.

Since 1939 the suit has been modified more than 250 times. As now manufactured, it comes in seven standard sizes which, by means of lacings and special zippers, fit every size and build of body. Doctor Francks is now a wing commander in the RCAF and has been decorated for his part in the invention of the suit. Sir Frederick Banting, his former colleague at the University of Toronto, was carrying data about the suit's construction and performance to England when he was killed in an airplane crash in Newfoundland in 1941.

The U. S. Navy closely followed the Canadians in its development work on G-suits. The Navy's design was modified by the American Air Forces after extensive centrifugal tests at the Aero Medical Laboratory at Wright Field, and within a relatively short time evolved the suit that is now standard for the AAF.

R. H. O.

Sea-Going Refrigerators for Pacific War Zones.—America's answer to the problem of how to provide her sons fighting in the far Pacific war zones, with fresh meats, vegetables and dairy products, has been the construction of a unique fleet of huge floating refrigerators.

Unlike any vessels ever built before, these sea-going ice boxes are either shipped, knocked-down, for assembly at island bases, or are towed across the Pacific. In addition to providing refrigerated space for storing and preserving perishable foods, a number of the vessels have equipment for making ice cream that will bring a touch of home to sweating jungle fighters.

Three of the ships—christened Hydrogen, Calcium, and Antimony—were built by Concrete Ship Constructors, National City, California. They are

built of concrete, with shells 7 to 9 inches thick, and have neither frames nor ribs. The hulls are 265 feet long, with a 48-foot beam and a 17-foot 7-inch depth. The deck house, of heavy wood construction, is divided into compartments, one of which is for making ice cream. Five hundred gallons of ice cream can be made daily.

The central compartment of the deck house is an enormous refrigerator, with walls heavily insulated with Fiberglas, and with massive doors. Below deck are 8 holds, all insulated with Fiberglas affixed to the concrete shell and the bulkheads. In these holds, temperatures of 10 to 15 degrees, Fahrenheit, can be maintained, even in the deepest tropics. Total cargo capacity of the refrigerated space is 1000 tons. The vessels are towed to their destinations.

A fleet of some 50 smaller refrigerator ships has been pre-fabricated by National Iron Works, San Diego, California. The vessels are shipped knocked-down, like the sections of a pre-fabricated house, for assembly at the bases to which they are consigned. Three hundred and sixty-eight individual packages are required for each vessel. Ten railroad box cars and gondolas are needed to move the 368 packages from the plant to the point of overseas shipment.

Hulls of these pre-fabricated refrigerator ships are 104 feet in length, with a beam of 29 feet and a depth of 8 feet. The hulls are divided into 4 holds, insulated with Fiberglas to aid in maintaining a temperature of 10 degrees, Fahrenheit. Cargo capacity of each vessel is 400 tons.

All of these unique ships will see front-line action in the Pacific war zones. Moved into position during a landing operation on a Japanese-held island, they will be camouflaged and serve as mobile refrigerators until permanent refrigeration equipment can be set up on shore. Then they will be moved to the next point of attack in the steady advance towards the home islands of Japan.

R. H. O.

Wood Laminating Program Has Played Important War Role.—More than two million feet of laminated wood is now in use in military vessels, the Office of Production Research & Development of WPB reports. This includes keels for 11,000 landing craft, keels and stems for 100 PT boats, and one-piece stem, keel, and stern post units for other ships.

These, as well as many other vitally important laminated items, have been produced by Gamble Brothers, Louisville, where a pilot plant was established two years ago. A second pilot plant has been set up at Timber Structures, Inc., Seattle.

The wood laminating program, which has meant so much in conservation of short supplies of wood and other materials, was conceived by the technical staff of the National Lumber Manufacturers Association and the Navy Bureau of Ships, and carried through with the cooperation of OPRD and the Forest Products Laboratory.

"Even our virgin forests never contained timber of the grade that has been supplied through laminating," said C. C. Dosker, vice-president of Gamble Brothers, describing the one-piece, curved, laminated stems and 65-foot, clear white oak keels supplied for PT boats.

"Skegs, or outer keels, for more than 11,000 landing craft have been made.

The Navy has stated that had it not been for laminating, the problem of procuring this material would have been literally an impossibility.

"Hundreds of thousands of laminated support staves for portable, canvas, water and fuel tanks are in service.

"More than 150,000 laminated paddles have been supplied the armed forces, not as a substitute, but as a better and stronger paddle than had been obtainable before.

"Thousands of spare parts in the form of laminated, heavy white oak timbers have been shipped to advanced repair bases in the European and Southwest Pacific theaters.

"Hammer boards, desperately needed by the drop forging industry, became a critical problem. That industry requires heavy sizes of hard maple, the best this country can produce. Laminating from one-inch lumber of lower grades resulted in a continued flow of forgings for tanks, shells, and other military requirements.

"Procurement of shuttle blocks, for which dogwood is the favored species, presented a major problem. Dogwood has never been too plentiful. We were able to demonstrate to the shuttle block manufacturers that, by laminating, the available supply of dogwood could be stretched and the exact sizes and kinds of shuttle block most needed could be produced."

The lamination program has been enlarged to produce structural parts for military truck bodies, mine guides, railroad trestles and highway bridges. The techniques learned in the necessities of war may be expected to carry over into the peace-time markets.

R. H. O.

Automobile Tire Production.—The rubber industry recently revealed for the first time that over 40 million heavy duty truck tires have come off production lines since Pearl Harbor.

This fact should give Tokyo no comfort. The Japs believed they had dealt a crippling blow to Allied war effort by grabbing over 90 per cent. of world sources of rubber. They knew we couldn't win on land or in the air without tires—millions of them.

These 40 million essential military and civilian heavy duty truck tires are one of the miracles of our come-back after the disaster at Pearl Harbor.

In addition to truck tires, the "shoes" of modern mechanized armies, the tire industry also reveals that it has boosted its production of tires for planes from 33,000 in 1939 to the huge total of 1,417,000 last year.

These figures are high-lights in a complete summary of tire production for the three years before and after Pearl Harbor issued by The Rubber Manufacturers Association, Inc.

"These facts about tire production to date," said Mr. A. L. Viles, President of The Rubber Manufacturers Association, "give perspective to the challenge which the industry faces today. The War Department asked tire manufacturers last December to increase the production of heavy duty truck tires by 75 per cent. They have accepted this challenge, regardless of cost or effort, and are well on the way to meeting it. Part of the challenge is to the public, however; a challenge to make their present passenger car tires last

through careful driving and re-capping while the industry concentrates on meeting these new military demands."

The key role which re-capping is playing in keeping civilian cars on the road is shown by the production figures on "camelback," the principal material used for re-capping. In 1941, 75,920,000 pounds of camelback were produced for re-cappers. Last year the industry produced more than four times that amount.

Behind all this record war production of tires is the amazing story of synthetic rubber, the creating overnight of a new industry to make a million tons of synthetic rubber a year. While the making of rubber from coal, petroleum and grains was being mushroomed from a laboratory process into huge synthetic rubber plants, the tire industry went ahead at full speed making the essential military tires from the precious stockpile of crude rubber which had been accumulated before Pearl Harbor.

The figures released by The Rubber Manufacturers Association show that U. S. stocks of crude rubber increased from 125,800 tons at the end of 1939 to a peak of 629,000 tons at the end of the first quarter of 1942. This unprecedented stockpile made it possible to keep our war forces supplied with tires while the new synthetic plants were being built and the industry was learning how to use the new material. By the start of 1943, substantial quantities of synthetic rubber were being used in making tires. In the first quarter of 1943, about twice as much crude rubber as synthetic was used in making tires. The proportion changed thereafter, more and more synthetic being used, until the figures for the third quarter of 1944 show 70,884 tons of synthetic being used as against 18,766 tons of crude.

An interesting wartime trend in tire production is revealed in the statistics showing the amounts of cotton and rayon used in building tires. Figures for the first quarter of 1942 show 43,631,000 pounds of cotton used and 5,936,000 pounds of rayon. In the third quarter of 1944, 45,240,000 pounds of cotton were consumed and 19,492,000 pounds of rayon.

R. H. O.

Signal Corps Institutes Relay Radio in France. (*Electrical Engineering*, Vol. 64, No. 2.)—A system of very high-frequency radio relay equipment has been introduced in France by the United States Army Signal Corps as an answer to the communications problem which arises when the stringing of wire cannot keep pace with the speed of the campaign.

This radio relay system consists of stations 25 to 100 miles apart, each beamed on the next. Police scout-car equipment which had been procured for expected police communications requirements in North Africa was found to be very well suited for radio relay systems. After the system was tested in North Africa, it was developed in the United States and England.

The actual combat-line communications are built around radio. In addition to the individual radio networks which each field unit has, every tank has a radio, and a certain number of infantrymen from each company are equipped with voice pack sets. For communication with the United States, a high-powered multi-channel 40-kw Army transmitter which took 25 days to assemble has been installed in France.

This Army radio station, with its $2,000,000 worth of equipment, operates

from batteries of radio and landline teletype machines and sends out approximately 400,000 words a day. Direct hookups with Washington, London, the Army world-wide radio communications system including Africa and Italy, and Army headquarters on the continent, make possible the transmission of impulses along supply lines over 4,000 miles long.

R. H. O.

Homing Pigeon's Reactions to Radio Waves Tested. (*Electrical Engineering*, Vol. 64, No. 2.)—Support for the belief that radio waves affect the instincts of homing pigeons has been supplied by a recent series of tests made by the United States Army Signal Corps. These tests apparently prove that radio transmission confuses the birds and retards them in fulfilling their flying missions.

Three successive tests were made with three different groups consisting of ten birds each, all of similar type and training. The birds were taken to a loft about ten miles from their home lofts. In each test five birds were released while the radio station was transmitting and the second five were liberated with the station silenced. Those birds released while the station was transmitting seemed completely bewildered. They circled erratically, very close to the station, for 15 or 20 minutes, then took off uncertainly for their lofts, requiring a total of 42 to 52 minutes to complete the 10-mile flight. Those freed while the station was silent made the usual brief circling, then took off promptly for the home loft with no confusion, covering the distance in 18 to 21 minutes. All birds flew under practically identical conditions of wind and weather.

The results of all three tests were virtually the same. However, the Signal Corps expects to test many more pigeons before the evidence can be accepted as conclusive.

R. H. O.

New Spray System for Doping Airplane Fabrics. (*Compressed Air Magazine*, Vol. 50, No. 3.)—Dope, the protective finish that serves to strengthen and stiffen airplane fabrics and to attach them to plywood plane sections, is usually applied cold by spraying. Today, through the development of what is known as the Thermotite System, the liquid can be applied hot. The equipment used consists essentially of a pressure tank, of a steam-heated reservoir, and of an air-driven circulating pump, all housed in a neat steel cabinet; of a spray gun; and of dual fluid hose lines, one to feed the dope to the nozzle and the other to return any surplus to the heater tank for reuse.

By the new method, a special full-body finish is put in the supply tank and is forced with air at 45 pounds pressure into the heater reservoir, where steam at $3\frac{1}{2}$ to 5 pounds pressure raises it to a predetermined temperature. From there it continues under pressure of the air to the spray gun, any excess liquid reaching the nozzle being by-passed to the heater shell by the circulating pump. The latter is driven at approximately 200 r.p.m., or at just enough speed to induce the suction necessary to overcome the pressure under which the dope is fed from the supply tank.

The Thermotite System eliminates the use of solvents to thin the dope for spraying, as heat changes its consistency from that of molasses to a free-flowing

liquid. In addition, it speeds the work because fewer coats are adequate. It is claimed that the finishing of wing or stabilizer surfaces, for example, which formerly involved as many as sixteen operations, now requires only seven.

R. H. O.

Transmitter-Receiver Designed for Lifeboat Use. (*Electrical Engineering*, Vol. 64, No. 3.)—A transmitter-receiver for use in lifeboats, equipped for two-way manual radiotelegraph and radiotelephone and for automatic radiotelegraph transmission, which can be operated by nonskilled men, has been announced by the United States Maritime Commission. The radiotelephone will enable occupants of lifeboats so equipped to maintain communications between lifeboats.

The transmitter-receiver will operate on pretuned international distress frequency of 500 kilocycles and on the high frequency adopted by the air/sea rescue program.

The transmitter-receiver is operable entirely by the use of a hand-powered generator. It is entirely waterproof and embodies the use of balloon and kite antennas enabling an average transmitting range ten times that obtainable with the present lifeboat transmitter on 500 kilocycles and a range above 1,000 miles when used on the high-frequency band.

R. H. O.

Timber Connector Highway Structures.—This is the title of a paper recently delivered before the Sixth Annual Highway Engineering Conference by Alden K. Smith.

Probably the greatest handicap of lumber has been the fact that it is easy to work. Arguments against its use stem, not from the shortcomings of the material itself, but usually from its misuse. However, when properly used, wood can be one of your most useful construction materials and economical in a wide range of highway structures.

Contributing to the usefulness of lumber for highway structures are devices used to increase loads transferred from one member to another, called timber connectors. Through their use in timber structures, design procedures have been simplified, and lumber has developed into an engineering material. Requiring less hardware and lumber, lower first costs as well as maintenance costs result. Time and labor are saved in erection, producing more economical structures with sound architectural lines.

The three types of timber connectors most generally used in timber highway structures are spike grids, split rings, and shear plates.

A spike grid consists of four rows of opposing spikes forming a $4\frac{1}{8}$ in. square grid with 16 teeth held in place by fillets. Flat grids are installed between frame trestle posts and braces. Single curve grids are used in pile trestle construction between the curved face of the pile and flat sawn bracing. This stronger and more rigid connection permits the trestle structure to act more nearly as a unit, thus reducing vibration and the possible loosening of joints. Wind loads, too, are easily carried through the spike grid joint. Grids are installed by being forced into the wood with a special inexpensive high strength rod and ball-bearing thrust washer assembly, or with hydraulic clamps. The grids have been carefully designed so that when installed in properly pressure-

treated lumber, the teeth penetrate only a small fraction of the depth of treatment, without harm to the preservative seal. Also, the pressure exerted on the wood by the teeth as they are inserted causes the treating fluid to flow around the teeth and completely fill any small checks which may develop.

The split ring, made in 2½ in. and 4 in. diameters, forms a mild steel circular band, beveled in its depth to permit easy insertion in the groove, and cut through at one place to form a tongue and slot. Placed in pre-cut circular grooves between adjacent faces of overlapping members, it provides a much larger bearing area for transferring loads than other methods of joining timbers. Tools for routing the grooves to receive split rings are operated by standard portable compressor or generator equipment. Split rings are used wherever two or more overlapping timber members come together in a joint. As an illustration, visualize a typical Howe truss with solid 12×12 chords and vertical tension rods. For split ring construction, the chord is separated into two 6×12's and the vertical may become a 4×12 instead of a steel rod. The end of the 4×12 is inserted between the two chord members, and split rings are installed in the overlapping faces. A direct transfer of stress is thus made between the chords and vertical through the split ring connectors.

Shear plates are manufactured in 2⅝ in. and 4 in. diameters, the smaller being pressed steel, the larger, malleable iron. Each is a circular plate with a flange around the edge at right angles to the face, extending from one face only, and with a central bolt hole. Shear plates are placed in pre-cut grooves or daps made by routing equipment similar to that for split rings. Developed primarily as aids to the demountability of timber structures, shear plates are widely used in strap and pin timber construction, gusset plate construction, foundation connections (all wood-to-steel connections), and wherever construction or erection joints require the sliding of one member between two others (wood-to-wood connections). Unlike spike grids and split rings which transmit loads directly from one member to another (the bolt merely holding the adjacent members in position), shear plates transfer loads through the bolts themselves.

Since the joint has commonly been known as the weakest part of any timber structure, the development of timber connectors has enabled engineers to design a multitude of economical timber highway structures from framed trestle bents fifteen feet apart to through trusses 210 feet in clear span and arches of 200 foot span. It is interesting to note here that the U. S. Navy used the arch principle, lumber and connectors, in building the largest clear-span timber structures ever built: gigantic blimp hangars 197 feet high, 1,000 feet long, and clear-spanning 237 feet! The War Production Board reported that such structures could not have been built of wood without the use of timber connectors.

Many timber bridge trusses, through and deck types, constructed along the Alaska Highway, serve to illustrate the scope of engineered timber roadway structures and the part played by timber connectors.

Early in 1942 the Public Roads Administration was handed the important job of providing stream and river crossings for the military road planned for construction through Canada and Alaska. Turning to one of our oldest construction materials, P.R.A. Engineers found ready and waiting an up-to-date system of construction advocated by the lumber industry and backed by

authoritative technical data and the vast resources of the forests themselves. With this relatively new, but well proven engineering data, and with the prompt support of industry technicians and experienced lumber fabricators, the Public Roads Administration quickly prepared a series of special and standard timber bridge designs, many of which, translated into actual construction, are now carrying important military traffic to and from the vast airfield network in Canada and Alaska. Simpler design methods, the basic engineering principles of the connector system of construction, and a well-established lumber fabricating industry enabled P.R.A. to perform, in record time and under tremendous handicaps, a miraculous construction feat.

Throughout the design of special and standard timber highway structures, the Public Roads Administration used its standard specifications, Form FP-41. Trusses were designed for an equivalent H-15 loading. Standard 24-foot roadways are provided and, on through structures, the overhead clearance is 14 feet.

R. H. O.

Casualty Blankets to Protect Wounded Airmen.—Through the recent announcement by the Automatic Blanket and Sunlamp Division of General Electric Co., Bridgeport, Connecticut, of a large quantity order for "casualty blankets," it became known that the U. S. Army Air Forces has decided to put at least one of these blankets in all heavy bombers as standard equipment.

Newly developed and tested under actual combat conditions in European theaters, the blanket, a wartime version of the peacetime automatic sleeping blanket, is being used for wounded crew members to prevent shock from excessive cold at high altitudes. It is already credited in army circles as having saved the lives of wounded flyers.

"As with the automatic sleeping blanket, the amount of heat put into the casualty blanket is determined by an automatic control invented by W. K. Kearsley of the General Electric Research Laboratory at Schenectady, New York," David C. Spooner, Jr., manager of General Electric's division making the casualty blanket for the Army Air Forces, said. "This control increases the energy input as the surrounding temperature goes down, and decreases it as the surrounding temperature rises with the descent of the bomber as it approaches its home base. In effect, it adds or removes blankets as required to give the wounded man or the sleeper just the right protection against the cold. The control developed for sleepers now has gone to war to care for the wounded."

The blanket consists of an electrically heated layer, well insulated with a wool fabric of thermal properties. It is lined with nylon to which a thin coat of neoprene has been applied. It will protect a man to temperatures as low as 60 degrees below zero.

R. H. O.

Structural Performance of Wartime Wood Buildings—and Its Postwar Application.—Frank J. Hanrahan, of the National Lumber Manufacturers Association, recently stated before the Illinois Society of Architects that in spite of the fact that many thousands of large wartime timber buildings were constructed under unfavorable circumstances, very few instances have been

reported where a roof truss actually came down. In a number of cases inadequate members were replaced or reinforced and bad construction details remedied. In many cases, stitch bolts or clamps were installed in or on checked or split members but frequently those installed were not needed.

Realizing that most of its wartime buildings were far below peacetime standards, the Army issued instructions that all its timber buildings on this continent be inspected and reported upon. As of last July, the Army had on this continent approximately 4,700 posts, subposts and outposts, comprising a floor area of over 1¼ billion square feet of buildings, which is equivalent to one building 92 feet wide reaching from New York to San Francisco.

The practically unanimous reactions of engineers, without and within the industry, who inspected the wartime timber structures were (1) amazement that many more failures did not take place in wartime structures in view of their abuse, and (2) a firm conviction that the design stresses in use prior to the war for stress-grade lumber were far too conservative, and therefore uneconomical, for timber buildings properly designed and constructed.

As a matter of fact, in August, 1943, the War Production Board issued a mandatory directive requiring an increase of 20 per cent. in working stresses for stress-grade lumber and in design loads for fastenings for timber buildings. Further, specific provision was made for the use of higher working stresses by the Army, Navy, or Maritime Commission to meet the objection that this increase of 20 per cent. was not large enough.

Pending a more exhaustive study, the U. S. Forest Products Laboratory subsequently recommended continuation of the use of the increased stresses of the W. P. B. Directive for postwar building construction.

After the amazing performance of wartime structures under adverse conditions, with the even greater efficiencies and economies to be realized from developments now in sight, and with the availability of reliable design specifications and standards, wood construction should play an even greater part in postwar building than it did in prewar building.

R. H. O.

Wolframite.—A possible new Alaskan supply of wolframite, a source of the metal, tungsten, used in toughening highspeed tool steel, has been discovered by the Geological Survey, Director W. E. Wrather of the Survey reported recently to Secretary of the Interior, Harold L. Ickes. The find was made during a geologic reconnaissance of the central Kuskokwim region on the northeast side of the Taylor Mountains, about 10 miles southeast of the Holitna river and 50 airline miles south of the village Sleitmut.

The specimen, a fragment of quartz containing crystals of wolframite about 2 inches long, was found in loose rock near the ridge crest west of a stream locally known as Stevens Creek, about 2½ miles from its head. The central mass of the Taylor Mountains is composed of coarse granite. The fragment containing wolframite appears to have broken from a quartz vein in a contact metamorphic zone about 3 miles wide formed in the sandstones and shales bordering the granite.

Lode wolframite has hitherto been found in Alaska only at Lost River on the Seward Peninsula and in a small vein on Deadwood Creek in the Circle district. Wolframite is found as float near Caribou Creek in the Bonnifield

district, and in placers at two localities, Deadwood Creek in the Circle district and Pearl Creek in the Fairbanks district.

R. H. O.

War Planes to be Housed in Glass Cloth Hangars.—Glass cloth, woven of glass fiber yarns and coated with either synthetic rubber or resin, has been selected by the U. S. Army Corps of Engineers for curtains, side walls and ends in newly developed airplane hangars installed at advance Army Air Forces bases.

Structural steel frame work and coated glass cloth are shipped to advance bases where the hangars are assembled for the A.A.F. Use of coated glass fabric for sections of hangars saves a great amount of shipping weight and speeds assembly of the hangars.

Two types of hangars incorporating the use of coated glass cloth have been designed by the Corps of Engineers for A.A.F. use.

One is a large hangar 162 feet long, 146 feet wide and 39 feet high which is capable of housing many planes. In this type hangar, coated glass cloth is used as a curtain or door across the entire 146-foot span.

Coated glass cloth is used for the side walls and end of a much smaller hangar known as the nose or shed type. Corrugated steel is used for the opposite closed end.

This type hangar is used primarily to house the forward part of planes. The fabric walls and end are of panel construction to allow the wing tips and tail of the plane to extend outside the hangar. This arrangement gives protective covering to the most vital parts of a plane.

Glass cloth was selected because of its resistance to the effects of mold and fungus, high strength in proportion to its light weight, and ease of handling to fit around plane fuselages. Canvas duck, previously used, was subject to rot from tropical dampness and would not withstand fungus attack. In Arctic regions, canvas would become semi-rigid due to the extreme cold. Glass cloth has good weathering properties, and is not affected by tropical heat or Arctic cold.

Coatings used on the glass cloth include neoprene and vinyl compound. Glass cloth is supplied by Owens-Corning Fiberglas Corporation, Toledo, Ohio.

R. H. O.

First Alcohol-From-Wood Plant Nears Completion.—Construction of one of the world's largest, and America's first, commercial plant for the production of ethyl alcohol from sawdust and waste wood by the Scholler-Tornesch method, has reached a point where it is confidently expected that it will be in operation by July 1st.

The $2,250,000 project occupies a 14-acre site at Springfield, Oregon. It is the property of the Willamette Valley Wood Chemical Company of Eugene, Oregon, which was formed by local lumbermen, themselves producers of large quantities of sawdust, the essential raw material. Financing was arranged through the Defense Plant Corporation.

An initial, daily consumption of 200 tons of sawdust with a corresponding output of 10,000 to 12,500 gallons of ethyl alcohol is planned, according to Charles Snellstrom, president of the company. The plant is designed to

increase that output greatly and ample supplies of raw material are available to permit rapid expansion.

The laboratory of the Oregon State College at Corvallis, under Dean Paul M. Dunn, is co-operating in a study of the qualities, deterioration under storage, and availability of wood waste. The relative qualities of planer shavings, hog fuel, sawdust and chipped slabs as potential raw material are being investigated.

Production on a commercial scale of ethyl alcohol, lignin, and other derivatives through the acid hydrolization of wood is new to the United States, but it is a well-established industry in Germany and one on which much of the German war economy has been founded. The German processes and patents are also being used by Italy and Japan.

Industrial alcohol in the United States has been derived in the past largely from blackstrap molasses brought here in tankers from the Caribbean. The requirements for industrial alcohol, multiplied by war, strained the capacity of the established facilities. At the same time the lumber industry was producing vast quantities of potential raw material for alcohol—sawdust—which has little or no commercial value in itself.

The Timber Engineering Company and the U. S. Forest Products Laboratory co-operated to adapt the German Scholler-Tornesch process to American conditions, the practicality of which was demonstrated in a pilot plant. The Willamette project is the realization by the lumber industry of another of the vast potentialities of the American forests.

R. H. O.

0 plus 1 equals 3.—Usually 1 plus 1 equals 2. But, for some insecticides, chemists have proved recently that 0 plus 1 may make at least 3.

Sesamine comes from sesame oil and will kill no flies. Pyrethrum kills many flies. Mix these two together, and the mixture will kill three to four times as many flies as pyrethrum alone. The practical result is a more deadly household fly spray now on the market. In this case, 0 plus 1 equals at least 3.

When we entered the war, U. S. Department of Agriculture chemists made use of just such a mixture in the "aerosol bomb." This bomb is a spray device now used by the armed forces to protect troops against disease by killing mosquitoes and flies in tents, airplanes, barracks and mess halls. It is a fortunate thing for the soldiers that some insecticides do not act according to arithmetic.

Chemists call this effect a synergy. Synergy means that two substances working together produce a greater effect than the sum of each when working separately. Applied to the farm, synergists increase the effectiveness of insecticides as they are used against crop pests. This saves time and money. It conserves materials. In the case of the aerosol bomb, the spray is not only more effective—it helps save more lives and better protects public health. After the war it may serve this same purpose in the farm home.

So far, synergists have been most practical in pyrethrum mixtures and against household and disease carrying pests such as the mosquito and housefly. The right combination of synergist with rotenone, nicotine, DDT, or other insecticides has not yet been found. Experiments by the Bureau of Entomology and Plant Quarantine are seeking synergists that will help farmers

combat crop pests at less cost. When some synergist makes 0 plus 1 equal 3 on the farm, it will mean considerable savings to farmers and more effective pest control.

R. H. O.

New Ordnance Weapons Announced.—(*Army Ordnance*, Vol. XXVIII, No. 149.) The Ordnance Department of the Army is at present directing over 1,300 major development projects on new and improved weapons. Col. S. B. Ritchie, assistant chief of the Ordnance Research and Development Service, has announced that approximately thirty new weapons are made ready for production each month.

"One of the outstanding accomplishments in shoulder arms of this war," Colonel Ritchie declared, "is the development of multiplicity of functions. New rifle grenades, for example, have been designed for anti-tank, anti-personnel, incendiary, smoke-screening and signaling purposes.

"The ammunition field has been greatly expanded and improved. A few items may be mentioned: anti-tank and anti-personnel mines (magnetic and non-magnetic) and their opposite mine-field clearing devices; sparkless-noiseless-flashless fuzes for grenades; and whole series of 20-mm. ammunition, including high-explosive, armor-piercing, and incendiary; the finest and most powerful demolition and fragmentation bombs in the world; new incendiary and chemical bombs; hyper-velocity ammunition (fitted with tracers so that the gunner can follow the path of the projectiles); a whole series of identification signals and flares for air and ground use; smoke ammunition in all calibers of field artillery; a combination of powder composition and primer to produce a minimum of smoke and no flash; the elimination of flash in big guns; a whole series of grenade signals and grenades, including smoke, fragmentation, and offensive types; a new type of concrete-piercing fuze which can be assembled to our standard high-explosive ammunition for artillery weapons in such a way that the projectile is transformed into a superior concrete-piercing shell, capable of blowing apart the strongest German pillboxes; mechanical delay and contact type bomb and artillery fuzes and other special fuzes; and illuminating shells for artillery and mortars of all types.

"The range of our artillery in general has been increased forty to fifty per cent over that of World War I.

"Miracles have been achieved in equipping our vehicles for flotation on the water and wading through deep water.

"The 90-mm. gun motor carriage M36 is an outstanding tank destroyer now being employed as effectively against German armor as in the days of the Saint Lo break-through when it knocked out on an average of ninety German tanks a day. Our M36 mounts a gun more powerful than the German 88-mm. gun and can out-maneuver all types of the relatively slow-moving German tanks. This 90-mm. gun, with its ammunition, is one of the most accurate weapons ever built in this country."

R. H. O.